TM211520TR **Reg. No :...............................**

Name :.................................

M. Sc. DEGREE (C.S.S.) EXAMINATION, NOVEMBER 2021

[2021 Admissions Regular and 2020 Admissions Improvement & Supplementary]

SEMESTER I - CORE COURSE (MATHEMATICS)

MT1C05TM20 - ORDINARY DIFFERENTIAL EQUATIONS

Time : 3 Hours **Maximum Weight : 30**

Part A

I. Answer any Eight questions. Each question carries 1 weight **(8x1=8)**

1. Let $u(x)$ be the nontrivial solution of $u'' + q(x)u = 0$ where $q(x) < 0$. Prove that $u(x)$ has atmost one zero.
2. Find the adjoint equation of $xy'' + (1-x)y' + py = 0$. Is this equation self adjoint? Why?
3. Show that $\frac{d}{dx}[xJ_1(x)] = xJ_0(x)$
4. Write the expression for n^{th} Legendre polynomial $P_n(x)$. Write first three Legendre Polynomials .
5. Find the value of $J_{\frac{-1}{2}}(x)$
6. Check whether the function $-x^2 - 4xy - 5y^2$ is positive definite, negative definite or neither.
7. Find the critical points of $\begin{cases} \frac{dx}{dt} = y(x^2+1) \\ \frac{dy}{dt} = 2xy^2 \end{cases}$. Solve the equation to find the path
8. Find the critical points of the system and solve the equation to find the path and sketch a few paths $\begin{cases} \frac{dx}{dt} = y(x^2+1) \\ \frac{dy}{dt} = -x(x^2+1) \end{cases}$
9. State Picard's Theorem. Why it is called a weak theorem.
10. Show that $f(x,y) = y^{1/2}$ satisfy a Lipschitz condition on the rectangle $|x| \le 1 \; and \; c \le y \le d, where \; 0 < c < d.$

Part B

II. Answer any Six questions. Each question carries 2 weight **(6x2=12)**

11. a) State Strum Seperation Theorem.

    b) Let $u(x)$ be the nontrivial solution of $u'' + q(x)u = 0$. Describe the location of roots of the solution $u(x)$ if $q(x) > 0 \; and \; q(x) < 0$
12. Find the eigen values and eigen functions for the equation $y'' + \lambda y = 0; y(0) = 0, y(2\pi) = 0$
13. Find the first three terms of Legendre series of $f(x) = \begin{cases} 0 & if -1 \le x < 0 \\ x & if \; 0 \le x \le 1 \end{cases}$
14. Prove that $e^{\frac{x}{2}(t-\frac{1}{t})} = J_0(x) + \sum_{n=1}^{\infty} J_n(x)[t^n + (-1)^n t^{-n}]$.

15. If the two solutions $\begin{cases} x = x_1(t) \\ y = y_1(t) \end{cases}$ and $\begin{cases} x = x_2(t) \\ y = y_2(t) \end{cases}$ of the homogeneous system $\begin{cases} \frac{dx}{dt} = a_1(t)x + b_1(t)y \\ \frac{dy}{dt} = a_2(t)x + b_2(t)y \end{cases}$ are linearly independent on [a,b]. Prove that $\begin{cases} x = c_1x_1(t) + c_2x_2(t) \\ y = c_1y_1(t) + c_2y_2(t) \end{cases}$ is the general solution of this homogeneous system on the same interval.

16. Find the general solution of $\begin{cases} \frac{dx}{dt} = x + y \\ \frac{dy}{dt} = 4x - 2y \end{cases}$

17. Describe Picard's method of successive approximation

18. Solve the initial value problem by Picard's method and compare the result with the exact solution

$$\frac{dz}{dx} = -y; z(0) = 0$$

$$\frac{dy}{dx} = z; y(0) = 1$$

Part C

III. Answer any Two questions. Each question carries 5 weight **(2x5=10)**

19. a) State and prove Strum Comparison Theorem.

    b) Find the normal form of Bessel's equation $x^2y'' - xy' + (x^2 - p^2)y = 0$ and use it to show that every non trivial solution has infinitely many positive zeros

20. a) State and prove the orthogonality properties of Legendre Polynomials

    b) Derive the formula for the coefficients a_n in the Legendre series.

21. Let $m_1 \; and \; m_2$ be the roots of the auxillary equation of the system $\begin{cases} \frac{dx}{dt} = a_1x + b_1y \\ \frac{dy}{dt} = a_2x + b_2y \end{cases}$. Explain the major and border line cases of the nature of roots.

22. Let $f(x,y) and \; \partial f / \partial y$ be continuous functions of x and y on a closed rectangle R with sides parallel to the axes. If (x_0, y_0) is any interior point of R then prove that there exists a number $h > 0$ with the property that the initial value problem $y' = f(x,y) \quad y(x_0) = y_0$ has one and only one solution $y = y(x)$ on the interval $|x - x_0| \leq h$.