TM211510TR **Reg. No :...............................**

Name :.................................

M. Sc. DEGREE (C.S.S.) EXAMINATION, NOVEMBER 2021

[2021 Admissions Regular and 2020 Admissions Improvement & Supplementary]

SEMESTER I - CORE COURSE (MATHEMATICS)

MT1C04TM20 - COMPLEX ANALYSIS

Time : 3 Hours **Maximum Weight : 30**

Part A

I. Answer any Eight questions. Each question carries 1 weight **(8x1=8)**

1. Verify that the point i is to the left of the positively oriented real axis and – i is to the right of the positively oriented real axes.
2. Define conformal mapping and give an example.
3. Prove that the reflection z $\rightarrow \bar{z}$ is not a linear transformation.
4. If a $\leq$ b, then show that $\left|\int_a^b f(t)\,dt\right| \leq \int_a^b |f(t)|\,dt$.
5. Evaluate $\int_{|z-a|=r} \frac{1}{z-a}\,dz$.
6. If f(z) and g(z) are analytic in Ω and if f(z) = g(z) on a set which has an accumulation point in Ω then show that f(z) is identically equal to g(z) in Ω.
7. Derive Cauchy's estimate.
8. Define residue of a function. Find the residue of f(z) = $\frac{1}{z^2 + 5z + 6}$ at its poles.
9. Explain (a) Homologous to zero (b) Locally Exact Differentials.
10. Find the residue of the function f (z) = $\frac{e^z}{(z-a)(z-b)}$ at its pole when a ≠ b and a = b.

Part B

II. Answer any Six questions. Each question carries 2 weight **(6x2=12)**

11. Prove that for a stereographic projection any circle on the sphere corresponds to a circle or a straight line in the z plane.
12. Prove that an analytic function in a region Ω whose modulus is a constant must reduce to a constant.
13. Show that the general line integral $\int_\gamma p\,dx + q\,dy$ depends only on the end points of γ iff the integral over any closed curve is zero.
14. Evaluate $\int_{|z|=2} \frac{dz}{z^2-1}$ for the positive sense of the circle.
15. Show that an analytic function comes arbitrarily close to any complex value in every neighborhood of an essential singularity.
16. If f(z) is analytic for |z| < 1 and satisfies the conditions | f(z) | $\leq$ 1, f (0) = 0, then show that | f (z) |$\leq$ |z| and |f' (0) | $\leq$ 1. Further, if |f(z) | = |z| for some z ≠ 0 or if |f' (0) | = 1 then show that f(z) = c z with a constant c of absolute value 1.

17. Let f(z) be analytic except for isolated singularities a_j in a region Ω. Then show that $\frac{1}{2\pi i}\int_\gamma f(z)dz$ = $\sum_j n(\gamma, a_j)\ Res_{z=a_j}$ f(z) for any cycle γ which is homologous to zero in Ω and does not pass through any of the points a_j.

18. Evaluate $\int_{-\pi}^{\pi} \frac{dx}{1+sin^2x}$

Part C

III. Answer any Two questions. Each question carries 5 weight **(2x5=10)**

19. (a) Discuss the uniform convergence of the series $\sum_1^\infty \frac{x}{n(1+nx^2)}$ for real values of x.

(b) Show that the cross ratio of four points is invariant under a linear transformation.

20. (a) State and prove Cauchy's theorem in a disk with exceptional points.

(b) Compute $\int_{|z|=1}|z-1||dz|$.

21. (a)Suppose that f(z) is analytic at z_o, f (z_o) = w_o and that f(z) - w_o has a zero of order n at z_o. If ε >0 is sufficiently small prove that there exists a corresponding δ >0 such that for all 'a' with $|a-w_o|$ < δ the equation f(z) = a has exactly n roots in the disk $|z-z_o| < \varepsilon$.

(b) State and prove the Schwarz's lemma.

22. (a) Use Residue theory to evaluate $\int_0^\infty \frac{x^2\,dx}{(x^2+a^2)^3}$.

(b) Evaluate $\int_0^\pi \log \sin\theta\ d\theta$.