TM211890TR

Reg. No :...............................

Name :.................................

M. Sc. DEGREE (C.S.S.) EXAMINATION, NOVEMBER 2021

[2021 Admissions Regular and 2020 Admissions Improvement & Supplementary]

SEMESTER I - CORE COURSE (APPLIED STATISTICS AND DATA ANALYTICS)

ST1C01TM - PROBABILITY AND MEASURE THEORY

Time : 3 Hours **Maximum Weight : 30**

Part A

I. Answer any Eight questions. Each question carries 1 weight **(8x1=8)**

1. Define Lebesgue measure. State the conditions under which a function is said to be Lebesgue measurable.
2. Prove that outer measure of a singleton set is zero.
3. If f and g are two measurable functions then show that f+g is measurable.
4. Let X be a random variable defined on the probability space $(\Omega, \mathscr{A}, P)$ and a and b are constants then show that $(aX+b)$ is a random variiable.
5. State continuity property of probability measure
6. State Tchebychev's inequality
7. State Markov's inequality
8. State Minkowski's inequality
9. Distinguish between Lindberg-Levy CLT and Lindberg-Feller CLT
10. state Liaponov's Central limit theorem

Part B

II. Answer any Six questions. Each question carries 2 weight **(6x2=12)**

11. Define sigma field. Give an example of a field but not sigma field.
12. If E_1 and E_2 are measurable sets, show that $E_1 \bigcup E_2$ is also measurable.
13. State Jordan decomposition theorem.
14. Show that $V(x) < E(x-c)^2$ for any $c \neq E(x)$
15. Define convergence is probability and convergence in distribution regarding a sequence of random variables {Xn, n ≥ 1} to a random variable X. Show that the former implies the latter
16. let $h(x)$ be a NOn-negative Borel measurable function of a random variable X and let $E(h(x))$ exists. then show that$p\{h(x) \geq \varepsilon\} \leq \frac{E[h(x)]}{\varepsilon}$ for every $\varepsilon > 0$
17. State and prove Lindberg-Levy central limit theorem
18. Let $\{X_n\}$ be a sequence of independent random variable with the following distribution .check whether does the Lindberg Condition hold?$\{^X n = \pm\ 2^n\} = \frac{1}{2}$

Part C

III. Answer any Two questions. Each question carries 5 weight **(2x5=10)**

19.
   1. If $\{f_n\}$ is a sequence of measurable functions which is fundamental in measure. Show that there exists a measurable *f* such that $f_n \rightarrow f$ in measure
   2. Let $f_n \rightarrow f$ in measure where *f* and each f_n are measurable functions. Show that there exists a subsequence $\{n_i\}$ such that $f_{n_i} \rightarrow f$ a.e.

20. State and prove

    1. Bayes theorem
    2. multiplication theorem

21. If Xn $\xrightarrow{p}$ X and Yn $\xrightarrow{p}$ Y then ST

    (a)Xn+Yn $\xrightarrow{p}$ X+Y

    (b)XnYn $\xrightarrow{p}$ XY

22. State and prove Kolmogorov's inequality