Project Report

On

A STUDY OF THE MODELLING OF CARBON DIOXIDE REMOVAL FROM THE ATMOSPHERE

Submitted

in partial fulfilment of the requirements for the degree of

MASTER OF SCIENCE

in

MATHEMATICS

by

POORNASREE RAMACHANDRAN

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Under the Supervision of

Smt.VEENA V S

DEPARTMENT OF MATHEMATICS

ST. TERESA'S COLLEGE (AUTONOMOUS)

ERNAKULAM, KOCHI - 682011

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ST. TERESA'S COLLEGE (AUTONOMOUS), ERNAKULAM

CERTIFICATE

This is to certify that the dissertation entitled, **A STUDY OF THE MODELLING OF CARBON DIOXIDE REMOVAL FROM THE ATMOSPHERE** is a bonafide record of the work done by Ms. **POORNASREE RAMACHANDRAN** under my guidance as partial fulfillment of the award of the degree of **Master of Science in Mathematics** at St. Teresa's College (Autonomous), Ernakulam affiliated to Mahatma Gandhi University, Kottayam. No part of this work has been submitted for any other degree elsewhere.

Date: 27/5/2022
Place: Ernakulam

Veena

Smt.Veena V S
Assistant Professor,
Department of Mathematics,
St. Teresa's College(Autonomous),
Ernakulam.

ST. TERESA'S COLLEGE
DEPT. OF MATHEMATICS
ERNAKULAM

Ursala

Dr.Ursala Paul
Assistant Professor,
Department of Mathematics,
St. Teresa's College(Autonomous),
Ernakulam.

External Examiners

1: DIVYA MARY DAISE S
27/05/2022

2: MARY RUNIYA
27/05/2022

DECLARATION

I hereby declare that the work presented in this project is based on the original work done by me under the guidance of **Smt.VEENA V S**, Assistant Professor, Department of Mathematics, St. Teresa's College(Autonomous), Ernakulam and has not been included in any other project submitted previously for the award of any degree.

Poornasree.

Place: **POORNASREE RAMACHANDRAN**

Date: 27/5/2022 **SM20MAT011**

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Poornasree

POORNASREE RAMACHANDRAN

SM20MAT011

Place:

Date: 27/05/2022

Contents

Chapter 1

INTRODUCTION

For the last few years, we people have been facing different kinds of natural phenomena globally.

The climatic conditions of our home earth are drastically changed, causing serious consequences to the humankind. The balance of our ecosystem is disrupted. Before anything worse happens, we have to make it stable.

The major situation we have to take care is, the rise in temperature of our atmosphere. The emission of global warming gases such as methane, carbon dioxide, etc. are the main reason for it. This can lead to adverse effects on humankind and our environment. Global warming may lead to severe problems like poor air quality, rise in sea levels, melting of glaciers, decrease in rainfall, draught, heat waves, effect on human health, etc.

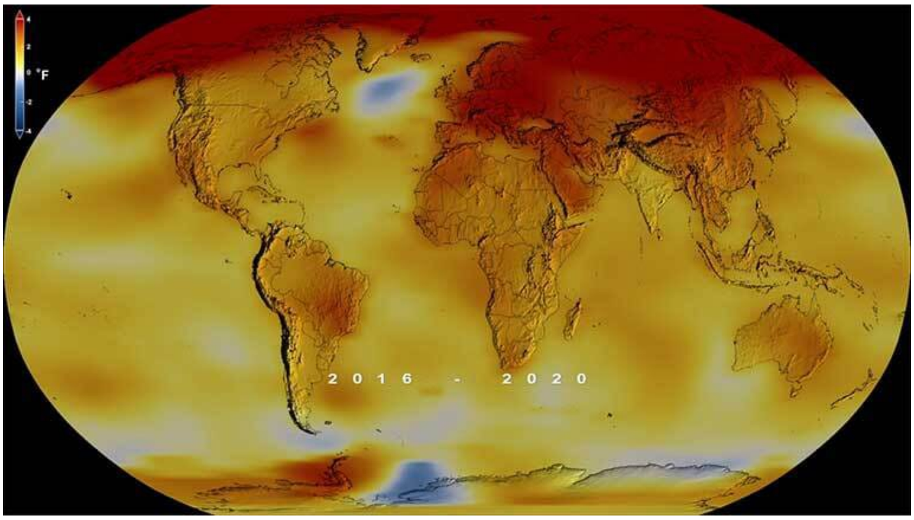
Carbon dioxide is the main contributor to global warming. In this project, we discuss the removal of carbon dioxide with the help of a mathematical model. And we discuss the different situations that may arise considering all of its related factors.

Chapter 2

BACKGROUND

According to the latest reports we got from NASA, 2020 was the hottest year on record. Also the average temperature of earth has risen significantly and it is due to the various human activities which include the emission of greenhouse gases like methane and carbon dioxide into the atmosphere.

We also know that our environment has its own capability of maintaining the equilibrium for the existence of life on earth. But unfortunately, the capacity of our ecosystems is becoming less efficient. They are not able to absorb the unwanted amount of such gases by themselves as earlier. So now it's our turn to find a solution to this. We should take necessary actions on time to avoid harsh consequences.



Chapter 3

TYPES OF MODELS

A Mathematical Model is a description of a system using mathematical concepts and language. Moreover, Mathematical Models are sets of equations that take into account many factors to represent a phenomenon. And the process of developing such a Model is termed Mathematical Modelling. Mathematical models are usually composed of relationships and variables. Relationships can be described by operators and Variables are abstractions of system parameters of interest that can be quantified. Several classification criteria can be used for mathematical models according to their structure:

- Linear vs. Non-linear
- Static vs. Dynamic
- Discrete vs. Continuous
- Deterministic vs. Stochastic

Chapter 4

FIELDS OF APPLICATIONS

- **Virtual reality**
- **Artificial intelligence**
- **Robotics**
- **Computer animation**
- **Astronomy:**
    - **(i) Detection of planetary systems**
    - **(ii) Origin of the universe**
    - **(iii) Evolution of stars**
- **Weather prediction**
- **Biology :**
    - **(i) Protein folding**
    - **(ii) Human genome project**
    - **(iii) Population dynamics**
    - **(iv) Evolutionary pedigrees**
    - **(v) Spreading of infectious diseases (AIDS)**
- **Flight simulation**
- **Air traffic scheduling**
- **Trajectory planning**

Chapter 5

CHEMICAL PROCESS

Carbon separation can be done in different ways. There are various methods like adsorption, absorption, membrane separation, oxygen combustion, sublimation and cryogenic separation.

In adsorption we use charcoal to capture carbon dioxide. In absorption we use a suitable absorbent of carbon dioxide and in membrane separation, we use different kinds of membranes to separate the carbon dioxide.

Here what we use is the absorption process and infusing liquid. In our case, we use a suitable absorbent and liquid species to reduce the rate of carbon dioxide in the atmosphere. What happens here is that, the liquid species and the absorbent gets reacted with the carbon dioxide in the atmosphere and forms another substance known as the secondary substance. And this secondary substance formed due to the reaction between carbon dioxide and other species is then removed by the gravity itself. And hence we can reduce the content of carbon dioxide present in our atmosphere.

5.0.1 SELECTED CHEMICAL PROCESS

The removal of Carbon Dioxide is done here by infusing liquid drops and suitable particulate matters (such as calcium oxide) in to the atmo-

sphere. When this global warming gas interacts with these externally introduced species, secondary phases are formed which are then removed from the atmosphere by gravity, reducing the concentration of global warming gas in the atmosphere.

Chapter 6

PROPOSED MODEL

We chose a place where the emission of the carbon dioxide takes place. There we introduce the liquid species and absorbent. To model the situation, we made some assumptions as follows;

(i) The amount of carbon dioxide emitted is constant.

(ii) The amount of liquid substance and absorbent used will be proportional to the amount of the carbon dioxide emitted.

(iii) The resultant substance formed due to the reaction between carbon dioxide and the substances we introduced will be removed from the atmosphere by the gravity itself.

(iv) There will always exists a threshold concentration of carbon dioxide in the atmosphere which cause no harm.

With the above assumptions, now let's define the variables needed for our model. Given below are the variables and what it denotes :

C – **the amount of carbon dioxide.**

Q – **the amount of carbon dioxide emitted from different sources.**

d_0C – **natural depletion of carbon dioxide.**

L – **the amount of liquid species.**

CP – **the amount of particulate matter formed.**

Ca – **the amount of a suitable absorbent.**

l_1CL – **depletion of co2 due to interaction with liquid species.**

l_0 – **natural depletion rate of liquid species.**

p – **the rate by which particulate matter is formed.**

p_0 – **natural depletion rate.**

a – **the rate of inflow of absorbent.**

a_0Ca – **natural depletion rate.**

a_1 – **interaction rate of carbon dioxide with absorbent.**

All the above constants are assumed to be positive.

The following is our system of equations which represents our model;

$$\frac{dC}{dt} = Q - d_0C - l_1CL - a_1CCa \tag{6.1}$$

$$\frac{dL}{dt} = l(C - C_0) - l_0L - l_1CL \tag{6.2}$$

$$\frac{dCP}{dt} = pl_1CL - p_0CP \tag{6.3}$$

$$\frac{dCa}{dt} = aC - a_0Ca - a_1CCa \tag{6.4}$$

Now using these, we are going to analyze three different cases. In first we will be using the liquid only, in the second we will be using the absorbent only and then in the third we use both the liquid species and absorbent together. After analyzing the corresponding equations and also analyzing with giving values to them, (i.e. Numerical valuation) we will find the case which gives the best result.

Chapter 7

ANALYSIS OF DIFFERENT SITUATIONS

7.1 CASE I

WHEN ONLY LIQUID SPECIES IS USED

Here the amount of carbon dioxide is greater than what should be maintained. That is, the amount of carbon dioxide in the atmosphere is more than its threshold concentration.

In this case we not the absorbent but only the liquid species to reduce the amount of carbon dioxide present. Now we form some algebraic equations by equating the first three equations from our system and form a final equation to analyze this case.

These are the equations taken from our system;

$$\frac{dC}{dt} = Q - d_0C - l_1CL - a_1CCa \tag{7.1}$$

$$\frac{dL}{dt} = l(C - C_0) - l_0L - l_1CL \tag{7.2}$$

$$\frac{dCP}{dt} = pl_1CL - p_0CP \tag{7.3}$$

The following are obtained from the above;

$$Q - d_0C - l_1CL = 0 \tag{7.4}$$

$$L = \frac{l(C - C_0)}{(l_0 + l_1C)} = f(c) \tag{7.5}$$

$$CP = \frac{pl_1CL}{p_0} \tag{7.6}$$

$$F(C) = Q - d_0C - l_1Cf(c) \tag{7.7}$$

From these equations it's clear that the amount of carbon dioxide is inversely proportional to the amount of liquid species too. Which means, as the amount of liquid species we use increases the concentration of carbon dioxide decreases.

7.2 CASE II

WHEN ONLY ABSORBENT IS USED

Here we check whether we can reduce the amount of carbon dioxide in the atmosphere by using an absorbent alone. We use an absorbent equal in proportion with the amount of carbon dioxide emitted.

To analyze this case we take the required equations from our system and equate it to zero to find the values of some variables and to obtain the required final equation.

Later we put values in the resultant equation to analyze it numerically.

Given are the equations from our system;

$$\frac{dC}{dt} = Q - d_0C - l_1CL - a_1CCa \tag{7.8}$$

$$\frac{dCa}{dt} = aC - a_0Ca - a_1CCa \tag{7.9}$$

From the above we obtained the following equations;

$$Q - d_0C - a_1CCa = 0 \tag{7.10}$$

$$Ca = \frac{aC}{a_0 + a_1C} \tag{7.11}$$

From these we can understand that as the interaction and rate of absorbent increases, the concentration of carbon dioxide decreases. It is inversely proportional.

We can also verify this by differentiating the concentration of carbon 'C' with the rate of inflow of absorbent and its interaction rate. In both cases we will be getting a negative value.

7.3 CASE III

WHEN BOTH THE ABSORBENT AND THE LIQUID SPECIES ARE USED

In this case too we assume that the amount of carbon dioxide present in the atmosphere is greater than what we need.

Here we use both the absorbent and liquid species together to reduce the concentration of carbon dioxide in the atmosphere. We form the required equations from our system which is already defined. Then we analyze the case. We will be checking whether this case removes or reduces carbon dioxide more efficiently than the rest. We will later check it numerically also.

The following are the equations from the system;

$$\frac{dC}{dt} = Q - d_0C - l_1CL - a_1CCa \tag{7.12}$$

$$\frac{dL}{dt} = l(C - C_0) - l_0L - l_1CL \tag{7.13}$$

$$\frac{dCP}{dt} = pl_1CL - p_0CP \tag{7.14}$$

$$\frac{dCa}{dt} = aC - a_0Ca - a_1CCa \tag{7.15}$$

The equations below are obtained from the above.

$$Q - d_0C - l_1CL - a_1CCa = 0 \tag{7.16}$$

$$L = \frac{l(C - C_0)}{(l_0 + l_1C)} = f(c) \tag{7.17}$$

$$CP = \frac{pl_1CL}{p_0} \tag{7.18}$$

$$Ca = \frac{aC}{a_0 + a_1 C} = g(c) \tag{7.19}$$

$$F(c) = Q - d_0 C - l_1 C f(c) - a_1 C g(c) \tag{7.20}$$

In this case, since we use both the absorbent and liquid species the result will be much effective. There may arise a situation where the natural conditions become unfavourable so the chances of getting an unexpected result is there when we use either absorbent or liquid species.

Here we have two factors to reduce the amount of carbon dioxide, so more the effect will.

The equation itself shows that the concentration becomes negative when we use more amount of absorbent and liquid species together. And a negative value of carbon means that the excess amount is removed.

Chapter 8

VARIATIONS OF PARAMETERS

Variation of L with Q

Differentiating 'L' with respect to 'C', we get;

$$\frac{dL}{dC} = \frac{ll_0}{(l_0 + l_1 C)^2} > 0 \tag{8.1}$$

And we already know that ;

$$\frac{dC}{dQ} > 0; \tag{8.2}$$

Therefore ;

$$\frac{dL}{dQ} = \frac{dL}{dC} . \frac{dC}{dQ} \Rightarrow \frac{dL}{dQ} > 0 \tag{8.3}$$

From this we can understand that as the amount of carbon dioxide emitted increases, the amount of liquid species we should use will also increases.

Variation of Ca with Q

Differentiating 'Ca' with respect to 'C', we get;

$$\frac{dCa}{dC} = \frac{aa_0}{(a_0 + a_1C)^2} \tag{8.4}$$

And we have ;

$$\frac{dC}{dQ} > 0 \tag{8.5}$$

Since

$$\frac{dCa}{dQ} = \frac{dCa}{dC}.\frac{dC}{dQ} \tag{8.6}$$

We get ;

$$\frac{dCa}{dQ} > 0 \tag{8.7}$$

From this we can understand that, as same as the above, here also, as the amount of carbon dioxide emitted increases, the amount of absorbent we should use will also increases.

Variation of C with l

Differentiating 'C' with respect to 'l', we get;

$$\frac{dC}{dL} = -\frac{l_1C^3}{Ql_0 + (l_1d_0 + ld)^2} < 0(8.8)$$

From this we can understand that the amount of carbon dioxide in the atmosphere decreases as the interaction between carbon dioxide and liquid species increases.

Chapter 9

NUMERICAL ANALYSIS

Now we are going to evaluate each of the equations in our cases by giving values to them numerically.

Let's first check what happens when only liquid species is used. And we can also verify whether the concentration of carbon dioxide is inversely proportional to the amount of liquid species as we got from the equations earlier.

For that;
Take the equation:

$$F(c) = Q - d_0C - l_1CL \tag{9.1}$$

Now let's fix the values of the variables other than 'L '.

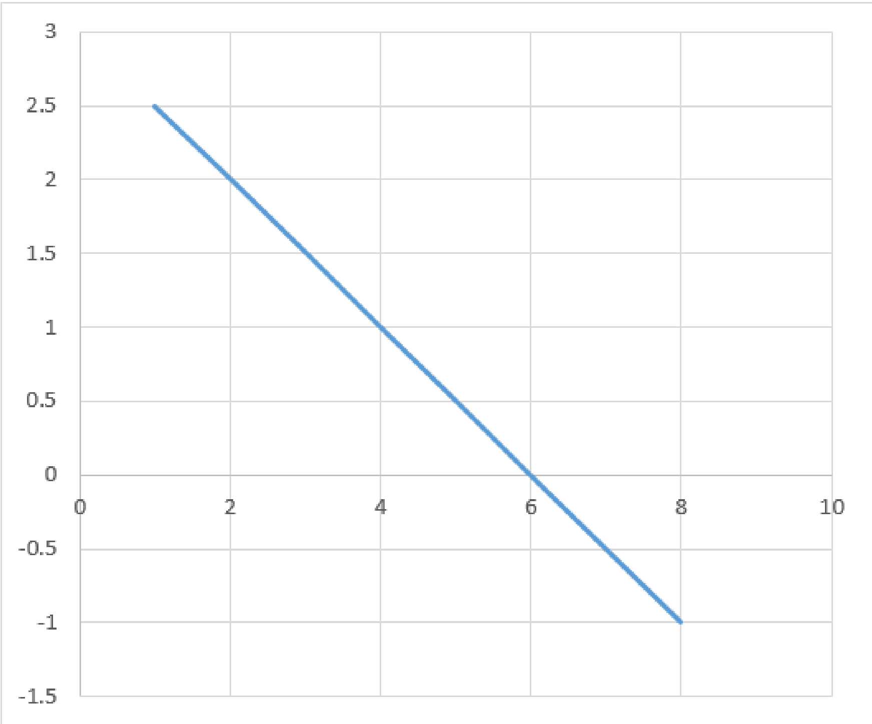
Let

$$Q = 4, d_0 = 1, C = 1, l_1 = 0.5 \tag{9.2}$$

From the graph below, clearly we can see that what we got from the equations is right. They are inversely proportional.

Table 9.1: Variations in the amount of carbon dioxide

x = L	y = F(C)
1	2.5
2	2
3	1.5
4	1
5	0.5
6	0
7	-0.5
8	-1



Now let's check the case when only absorbent is used.

Take the equation:

$$F(c) = Q - d_0C - a_1CCa \tag{9.3}$$

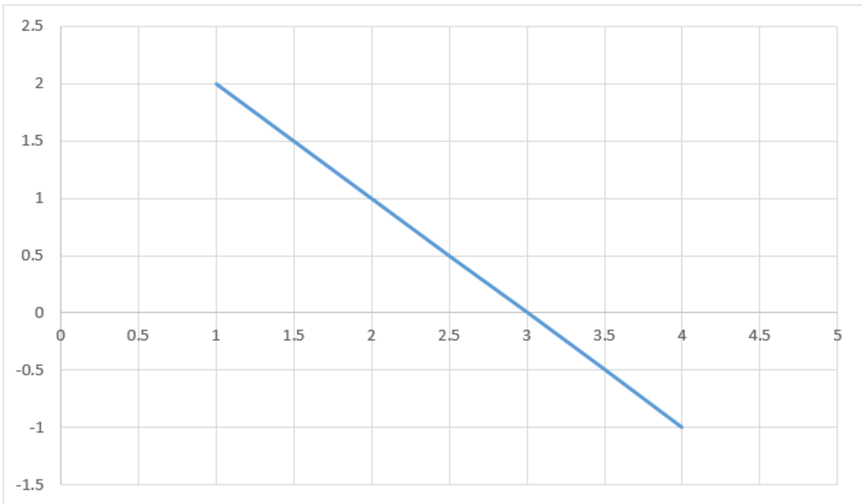
Now fix the values of the variables other than 'Ca '.

Let

$$Q = 4, d_0 = 1, C = 1, a_1 = 1. \tag{9.4}$$

Table 9.2: Variations in the amount of carbon dioxide

x = Ca	y = F(C)
1	2
1.5	1.5
2	1
2.5	0.5
3	0
3.5	-0.5
4	-1
4.5	-1.5



In this case also the concentration of carbon dioxide is inversely proportional. As the amount of absorbent increases the amount of carbon dioxide decreases.

Now let's check the case when both absorbent and liquid species is used.

Here we fix the values of variables except for 'L' and 'Ca'.

Let

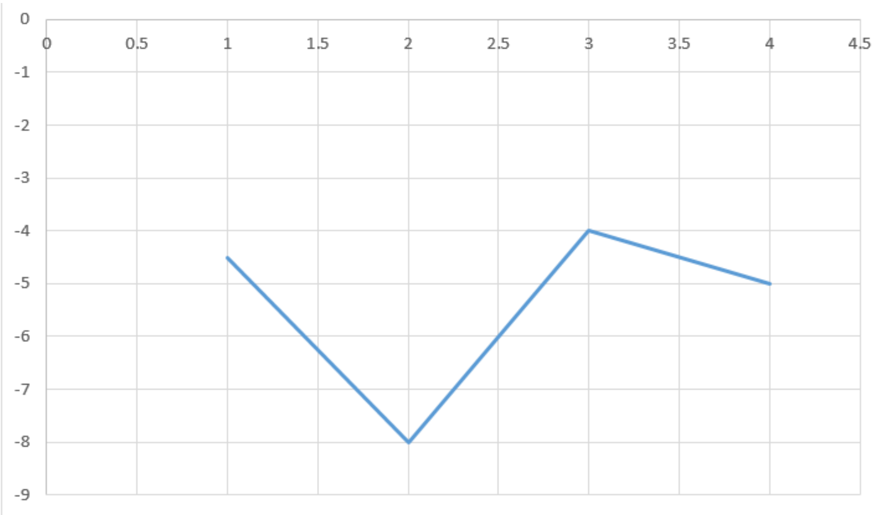
$$F(c) = Q - d_0C - l_1CL - a_1CCa \tag{9.5}$$

and the values be;

$$Q = 4, d_0 = 1, C = 2, l_1 = 1.5, a_1 = 1. \tag{9.6}$$

Table 9.3: Variations in the amount of carbon dioxide

x=L	y=Ca	F(C)
0.5	2	-3.5
1.5	1	-4.5
2	2	-8
1	1.5	-4
2	0.5	-5



Here we can see that the graph is completely in the negative region.

Chapter 10

CONCLUSION

After checking all the situations, we understood that the concentration of carbon dioxide decreases as the amount of the species we used for the reduction of carbon dioxide increases. The amount of carbon dioxide can be reduced and maintained in all cases. But we have to find the better one. That is, the case which is more efficient.

Since there is a chance for the absorbents to not react as we expect, it is better to use both liquid species and absorbents together. That is, the case III will yield better results than the other two. And we should always introduce the absorbents near the place where the emission exactly takes place so that the reactions will be effective. Timely action must be taken for the removal of carbon dioxide, or otherwise, it will cause a dangerous impact on the life on earth.

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