ST. TERESA'S COLLEGE (AUTONOMOUS), ERNAKULAM

CERTIFICATE

This is to certify that the dissertation entitled, **INTERVAL GRAPHS, TOLERANCE GRAPHS- IT'S APPLICATIONS** is a bonafide record of the work done by Ms. **MARY IMELDA** under my guidance as partial fulfillment of the award of the degree of **Master of Science in Mathematics** at St. Teresa's College (Autonomous), Ernakulam affiliated to Mahatma Gandhi University, Kottayam. No part of this work has been submitted for any other degree elsewhere.

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DECLARATION

I hereby declare that the work presented in this project is based on the original work done by me under the guidance of **DHANALAKSHMI O M**, Assistant Professor, Department of Mathematics, St. Teresa's College(Autonomous), Ernakulam and has not been included in any other project submitted previously for the award of any degree.

Ernakulam.

Date: 27-05-2022

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ACKNOWLEDGEMENTS

I must mention several individuals who encouraged me to carry this work. Their continuous invaluable knowledgably guidance throughout the course of this study helped me to complete the work up to this stage

I am very grateful to my project guide (Dhanalakshmi O M) for the immense help during the period of work

In addition, very energetic and competitive atmosphere of the Department had much to do with this work. I acknowledge with thanks to faculty, teaching and non-teaching staff of the department and Colleagues.

I also very thankful to HoD for their valuable suggestions, critical examination of work during the progress.

Ernakulam.   MARY IMELDA

Date: 27-05-2022   **SM20MAT010**

Project Report

On

INTERVAL GRAPHS, TOLERANCE GRAPHS- IT'S APPLICATIONS

Submitted

in partial fulfilment of the requirements for the degree of

MASTER OF SCIENCE

in

MATHEMATICS

by

MARY IMELDA

(Register No. SM20MAT010)

(2020-2022)

Under the Supervision of

DHANALAKSHMI O M

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ERNAKULAM, KOCHI - 682011

APRIL 2022

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Contents

Chapter 1

Introduction

In graph theory, an interval graph is an undirected graph formed from a set of intervals on the real line, with a vertex for each interval and an edge between vertices whose intervals intersect. It is the intersection graph of the intervals. Interval graphs have a variety of applications in various fields such as radio frequency allocation, VLSI architecture, temporary thinking in AI, storage allocation, used for modeling food webs, and studying planning problems etc. Also a compilation of intermediate graphs, called Tolerance graphs was introduced in 1982 by Columbus and Monma. Several uses are proposed in the fight against the COVID-19 epidemic since graphs appear as a natural tool in shaping the various problems associated with this global epidemic by slowing down the process of transmitting the virus. This method can be used to fight other epidemics as well. In this project, I've tried to emphasize the importance of interval graph by discussing it's properties and thus applications too and also solved a flight scheduling problem by finding out the minimum number of gates required in a day considering the timetable of departure flights of domestic as well as international schedules. It is done using the Mathematical Software named 'Sage-Math'.

Chapter 2

PRELIMINARIES

A Graph is an ordered pair (V,E) where V is the set of vertices and E, the set of edges connecting the vertices.

Transitive Orientation: Obeys the transitive property: if an edge ab is oriented from vertex a towards vertex b, and edge bc is oriented from b to c, then there is an edge ac and it is oriented from a to c.

Intersection Graph: An intersection graph G is an undirected graph formed from a family of sets Si , i = 0, 1, 2, ... by creating one vertex vi for each set Si, and connecting two vertices vi and vj by an edge whenever the corresponding two sets have a nonempty intersection.

Chord: A cycle has a chord if there are a pair of vertices that are adjacent, but not along the cycle.

Perfect Graphs: In perfect graphs, chromatic number of every induced subgraphs equals the order of largest clique of that subgraph.

Chromatic number: Smallest number of colors needed to color a graph.

Induced Subgraph: Graph formed from a subset of vertices of the graph and all of the edges connecting pairs of vertices in that set.

Clique: A set C is a clique of the graph F, iff C is a subset of V(G) and u,v in C where u v such that uv belongs to E(G).

Maximal Clique: Clique that cannot be extended by adding in another vertex of the graph.

2.1 SageMath

Originator and Leader of Sage is William Stein, a mathematician at University of Washington. It was first released in 2005. SAGE- System for Algebra and Geometry Experimentation. Sage is a free, open source Mathematical software that supports research and teaching in algebra, geometry, number theory, cryptography etc... Both Sage development model and Sage technology itself is divided by the greatest emphasis on openness, community, cooperation and collaboration. The ultimate goal of Sage is to create a functional, free, open source different from Maple, Mathematica, Magma, and MATLAB.It builds on top of many existing open-source packages like NumPy, SciPy, Maxima, GAP, R(Statistics) and many more. Python programming language is the backbone of SageMath.

SageMath Operations:

*Running SageMath: Type any SageMath command in the input box, and press Shift+Enter to see the result.

*Graph(5): Build a graph on 5 vertices

*Graph0:[1,2], 1:[2,3], 2:[3] is a graph 4 vertices, where vertex 0 is connected to 1,2; 1 is connected to 2,3; and 2 is connected to 3.

*return: the result of the function is given by return.

Chapter 3

INTERVAL GRAPHS

3.1 Origin

In 1957, a Hungarian mathematician named Hajos introduced the Interval Graphs by examining the Intersection of Intervals. About the same time, the American biologist Seymour Benzer was looking at genetic problems, especially studying the structure of fragmented genes. In his 1959 paper, he speculated that the fragments were straightforward pieces, and he also asked about the interlocking structure of the intersection lines, and how they would pass. Hajos watched from time to time; Benzer was looking at genetics. Benzer initiated the study of intervals in his paper which included the study of structure of bacterial genes. At that time it was not clear whether the DNA of bacterial genes are linear or not, but Benzer's work was the basic in establishing the fact that the genes are linear. He obtained data on the overlap of fragments of the gene and showed the data consistent with linearity. Interval graph arise in connection with the restriction map that shows certain site's location on specific DNA.

3.2 Definition

Interval graphs are the Intersection graph of intervals on real line, where each vertex represents an interval and two vertices are adjacent if their corresponding intervals intersect. G=(V,I) is an Interval Graph, if we can associate a set of Intervals I=(I1,. . . ,In) on a line where V=(1,. . . ,n) and two vertices, x and y, are linked by an edge if and only if their intervals have non empty intersection.

3.3 Properties of Interval Graphs

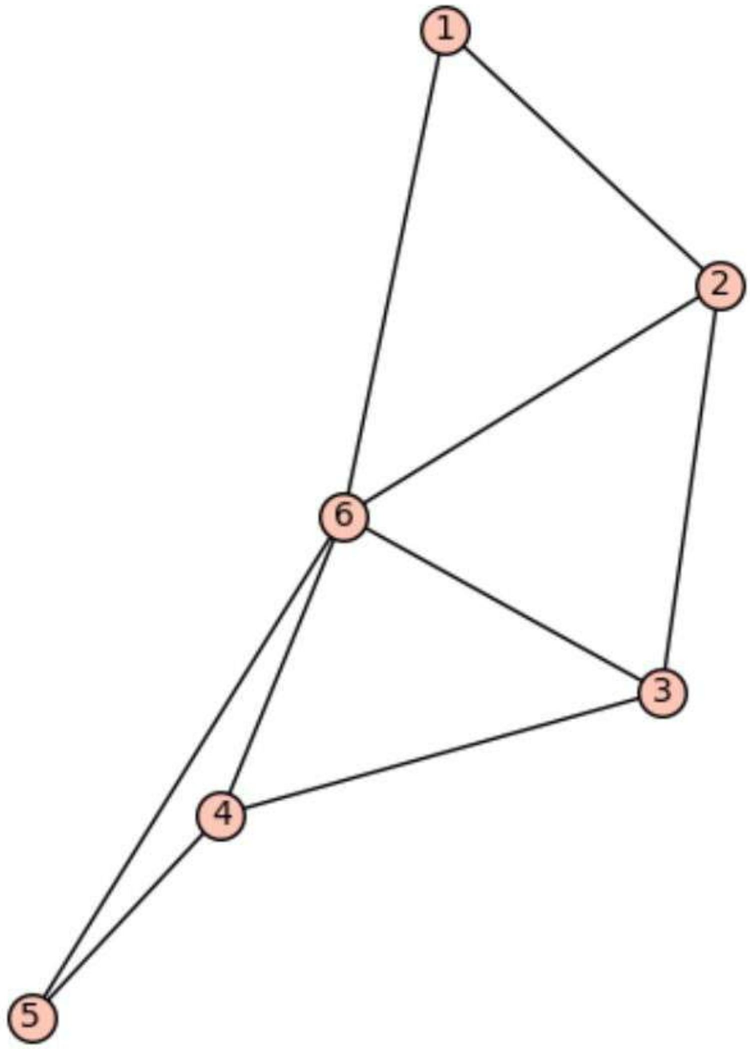

Figure 3.1: Interval Graph

3.3.1 Chordal Graph Property

Interval Graphs are Chordal Graphs, that is every cycle of length greater than or equal to 4 will have a chord in it;which means there will be an edge connecting two vertices that are not consecutive in the cycle.

3.3.2 Perfect Elimination Scheme exist

A graph is an interval graph iff a perfect elimination scheme exist. If v1,v2,... vn are the vertices, then in perfect elimination scheme ; considering v1, all the vertices that is adjacent to v1 will be connected each other. Then if we remove v1, then again considering v2 all the vertices adjacent to v2 will be connected to each other and so on... We continue this until the remaining graph is empty.

3.3.3 co-TRO property

The complement of Interval Graph satisfies Transitive Orientation.

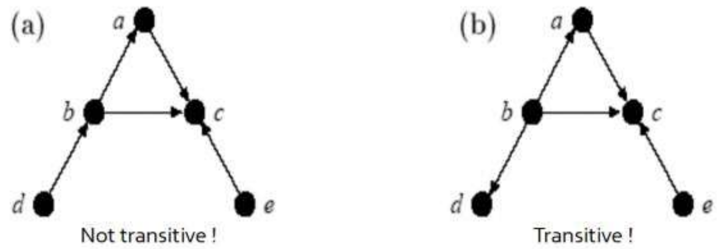

Figure 3.2: co-TRO property

3.3.4 Perfect Graphs

Interval graphs are perfect graphs. In perfect graph, chromatic number of the graph will be equal to its clique number.

3.3.5 Consecutive Ones Property

Consecutive ones property is an important property of the Interval graphs. From an interval graph(or Intersection graph), we can construct a binary matrix(with 0's and 1's) which is also called the clique matrix. This matrix is said to satisfy consecutive one's property for columns if its rows can be permuted in such a way that 1's in each column should occur consecutively.

Chapter 4

APPLICATIONS OF INTERVAL GRAPHS

4.1 Grocery Shop Problem

Several People spend Saturday morning at a grocery shop. If 2 people are there at the same time, they can see each other. -Adam arrives at 9.00am and stays for 2 hours -Bob arrives at 9.45am -Carol arrives with Adam and leaves with Bob, before Adam is ready to leave. -Dave arrives as Adam is leaving and leaves at noon. -Ella spends the shortest amount of time, but she sees Adam, Bob, Carol, Dave. -Frank sees Adam, but not Bob How many people could Bob see at the grocery shop? Figure 4.2 is the Interval graph corresponding to the

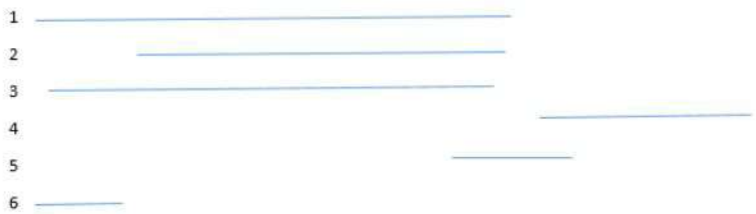

Figure 4.1: Grocery shop problem-Intersection graph

given problem. Here vertices represents each person (1-A,2-B,3-C,4-D,5-E,6-F) and the vertices are named from the starting letter of their names (A, B, C, D, E, and F) Figure 4.1 represents the Intersection Graph. From the figures, it is clear that, Bob could meet with 3 people at the grocery shop. This can also be considered as an application of

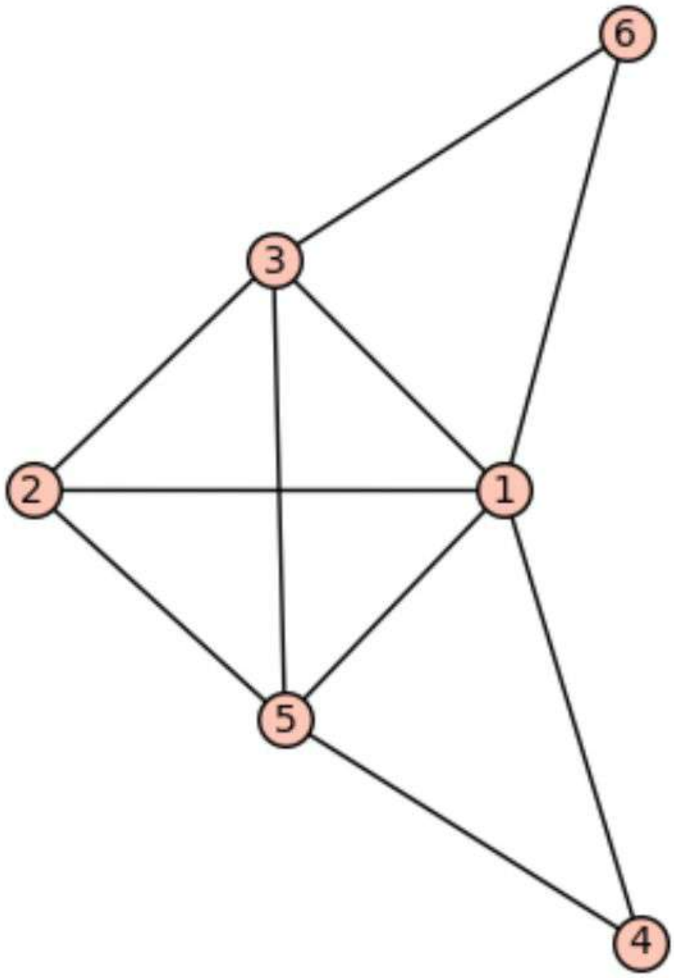

Figure 4.2: Grocery shop problem-Interval graph

the current pandemic situation. If we need to find the primary and secondary contacts of a person who may have affected with the virus, then it can easily be found from an interval graph.

4.2 Room Scheduling Problem

For a particular day, in a university there are a number of lectures scheduled and their time schedules are given as follows: Lecture a may go from 9:00 to 10:15, Lecture b may go from 10:00 to 12:00, etc.. (refer Fig 4.3) Conflicting lectures require different rooms; that is, when two time intervals overlap, we cannot schedule those lectures to the same room, as is the case for Lectures a and b. What is the minimum number of classrooms required to schedule the full set of lectures?

Figure 4.3 illustrates such a scheduling problem with six lectures.

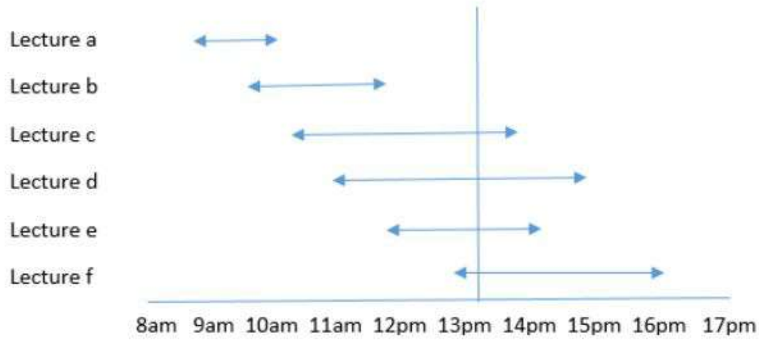

Figure 4.3: Room scheduling problem

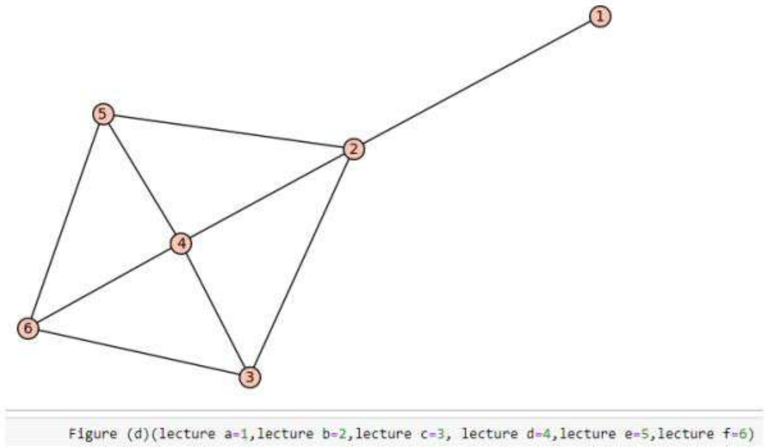

Figure 4.4: Room scheduling problem-Interval graph

Figure 4.4 shows the corresponding interval graph. The vertical cut at 13:15(figure 4.3) indicates a time when four classes are "alive" at the same moment, implying that we will need at least four classrooms. Also from the corresponding interval graph we can observe that the number of vertices of the maximum clique is 4. Hence the answer is 4.

4.3 Illustration of Consecutive Ones Property- Biological Application

Consider a hypothetical dataset with eight overlapping fragments (I1 to I8) as the intervals. Their pairwise overlap is considered and an adjacency matrix is constructed. Figure 4.5 shows the adjacency matrix where entry in the ith row and jth column is 1, if the vertices i and j are adjacent (fragments overlap) and 0 otherwise. The adjacency matrix is a square symmetric matrix. Figure 4.6 shows the corresponding intersection graph, called the interval graph. Each interval corresponds

1	1	2	3	4	5	6	7	8
2	1	1	0	0	0	0	0	0
3	1	1	1	1	1	1	0	0
4	0	1	1	1	1	0	0	0
5	0	1	1	1	1	1	0	0
6	0	1	0	1	1	1	0	0
7	0	0	0	0	1	0	1	1
8	0	0	0	0	0	0	1	1

e.)Adjacency Matrix

Figure 4.5: Adjacency matrix

to a vertex and two vertices u and v are connected with an edge if and only if the intervals u and v overlap.(This interval graph is constructed using SageMath) The corresponding 'clique matrix' for the figure 4.7

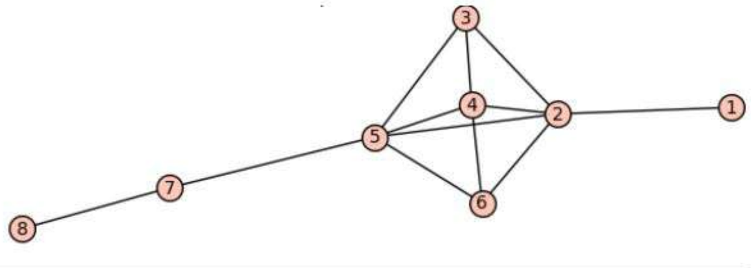

Figure 4.6: Interval graph

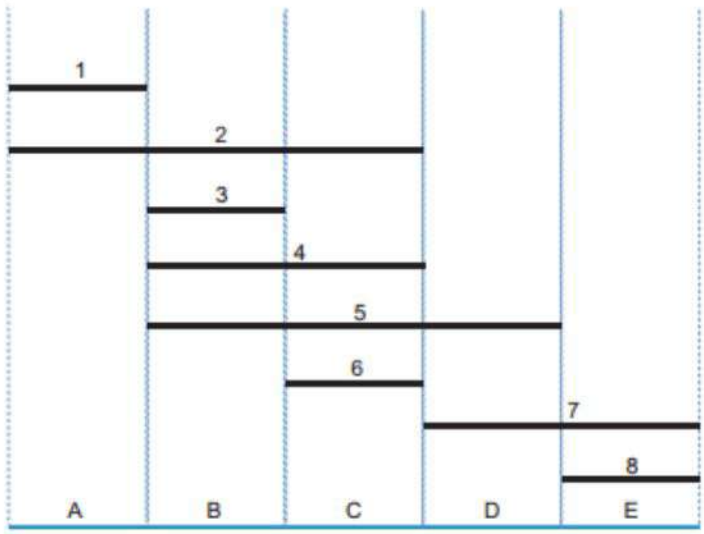

Figure 4.7: Intersection graph

is given below. This matrix makes it clear the consecutive ones property, that is 1's in each column occur consecutively. Considering figure 4.7,maximal cliques are A, B, C, D, E and they are ordered in a way. For example : the maximal cliques containing I2(interval 2) are in a order, that is A, B, C. Similarly, that of I5 are B, C, D also occur in a

	1	2	3	4	5	6	7	8
A	1	1	0	0	0	0	0	0
B	0	1	1	1	1	0	0	0
C	0	1	0	1	1	1	0	0
D	0	0	0	0	1	0	1	0
E	0	0	0	0	0	0	1	1

Figure 4.8: Clique matrix

sequence and so on.

4.4 Berge Mystery Problem

A murder mystery, original version authored by Claude Berge in 1994. Six professors had gone to the library on the day when a rare tractate was stolen. Each one came in once, stayed for a while and then left. If two were in the library at the same time, at least one of them saw the other. Investigators questioned the professors, since they were trying to identify the person who had stolen and they collected the following informations:

* Abe said that he saw Burt and Eddie. * Burt reported that he saw Abe and Ida. * Charlotte claimed to have seen Desmond and Ida. * Desmond said that he saw Abe and Ida. * Eddie testified to seeing Burt and Charlotte. * Ida said that she saw Charlotte and Eddie. But one of the professors lied, who was it? In figure 4.9, the set of facts is shown in the form of a digraph, in which each of the pointed edges shows a "true" statement: who said who they saw. Now, double-edged arrows immediately mean "truth", for example, Burt said he saw Abe and Abe saying he saw Burt, so even if one of them lied and the other

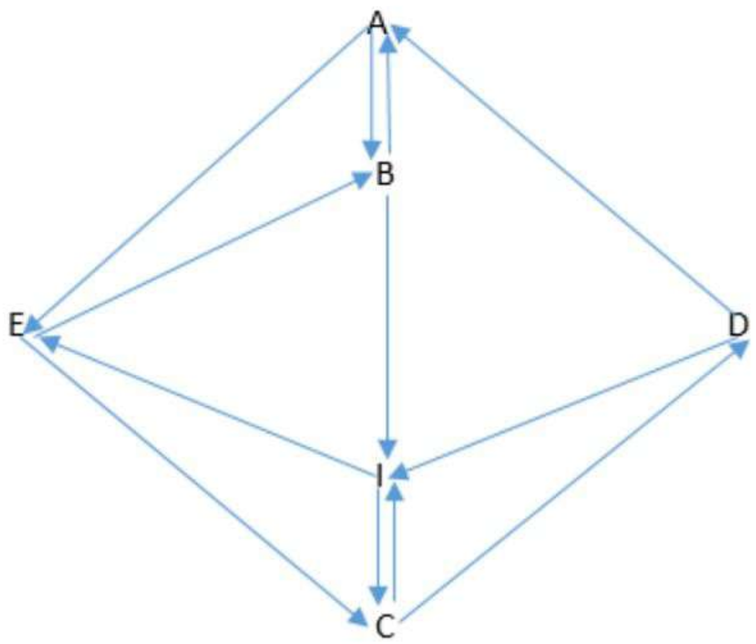

Figure 4.9: Berge Mystery Problem

told the truth, they were actually in the library at the same time. In the case of Ida and Eddie, however, maybe they were in the library or maybe not depending on whether Ida is the liar.

Solution using Interval Graph (property):

We can see a chordless cycle in Figure 4.9, cycle A, B, I, D suggest that one of these professors is a liar, because if all these people were telling the truth, we would be getting a contradiction with the property that the interval graph does not have a chordless cycle! This observation will help to solve the problem: Since Charlotte and Eddie did not participate in cycles A, B, I, D, they must have told the truth, because the liar is one of four in this cycle. But it is not the end of the cycle around here, for example, there is a large chordless cycle that goes everywhere around A, E, C, D, which means that Burt and Ida also told the truth. So now we are under two liars, Abe or Desmond. If Abe is a liar and Desmond is right, then A, B, I, D would remain a chordless 4-cycle since B and I are truthful. Therefore, Desmond is the liar.

Chapter 5

TOLERANCE GRAPH AND IT'S APPLICATIONS

This class of graphs was introduced in 1982 by Martin Charles Golumbic and Clyde Monma at the 13th Southeastern Conference on Combinatorics, Graph Theory and Computing held at Boca Raton who used them to model scheduling problems in which the tasks to be modeled can share resources for limited amounts of time, also to generalise some of the well known applications of Interval graphs. A graph G = (V, E) is a tolerance graph if each vertex v in V can be assigned a closed interval Iv and a positive real value called tolerance tv , such that xy belongs to E if and only if $\mid Ix \cap Iy \mid \geq min(tx, ty)$.

A collection (I, t) of intervals and tolerances is called a tolerance representation.

If we restrict all the tolerances tx to be equal to any fixed positive constant c, then we obtain exactly the class of interval graphs.

5.1 Different types of Tolerance Graphs

5.1.1 Bounded and Unbounded Tolerance Graphs

A tolerance graph is bounded tolerance graph if it admits bounded tolerance representation and if otherwise it is unbounded. .

5.1.2 Proper Tolerance Graph

If a graph G has a tolerance representation such that no interval properly contains another interval, then G is called proper tolerance graph.

5.1.3 Unit Tolerance Graph

If graph G has a tolerance representation such that every interval is of unit length, then G is a unit tolerance graph.

5.2 Properties

An important property of Tolerance graphs are that they are perfect graphs. For any tolerance representation, any one or all of the following 5 properties are satisfied: a.)Tolerances are strictly positive b.)Tolerances are all distinct(except to that set to infinite) c.)The end points of Intervals are distinct d.)Intersection of the Intervals are non empty e.) Any tolerance which is larger than the length of its corresponding interval is set to be infinity; such a tolerance is called unbounded (mentioned above)

Regular Representation: Tolerance representation satisfying all the above 5 properties.

5.3 Application of Tolerance Graph

The following application of tolerance graph can be used to combat Covid-19 pandemic (or it is also applicable for other infectious diseases too), by limiting the spread of the disease. From 2020, this pandemic is causing terrible loss of lives and also it has a great impact on the economy of all over the world. Many measures are taken to fight this pandemic. Mathematics and computer science can also play an important role in this. Graphs appear as a natural tool in various problems related to the COVID-19 epidemic, making use of artificial intelligence; for example searching for infected contacts, to do other medical analyzes to understand the virus and the potential for disease, and, in general, to predict the dynamics of Covid-19 pandemic. Here in particular we will be dealing with the application of Tolerance graph for the same. One of the most important things an epidemiologist can diagnose to reduce the spread of the virus is to determine events that can be called critical, which means many people can get infected at these events and hence they can give advice to minimise or restrict the number of participants in those events. This problem can be modeled as follows. Let G be the graph of their set of vertices corresponding to the set of intervals I = I1,. . . , Iv. Each vertex represented by Ij = [sj, ej], where sj and ej are initial time the person j get infected and the ending time of infection respectively. At each vertex, we add tolerance, ti, i = 1,. . . , v.

5.3.1 Scheduling Flights

One of the real life problems that can be modeled with tolerance graphs is connected to flight schedule during the pandemic. I have considered application of the algorithm to the departure flights scheduled at the Cochin International Airport for August 23,2021 . The schedule was found at their website:

https://cial.aero/flightstatus/status.aspx?type=
DEPARTURElinkLvl1Id=8linkId=10.

This airport has a rule that passengers use domestic flights must be at the gates 2 h before flight, and those for international flights 3 h earlier. To minimise the spread, it is needed to schedule flights a and b for different gates if period of time of the passengers in these two flights should be at the gate overlap for more than 30 min. Figure 5.1 gives the information of the timing of all the flights scheduled for a day(23 Aug 2021) in Cochin International Airport. If the flight is domestic, then let Ij = [sj - 2, sj] and Ij= [sj- 3, sj] if it is international, where sj is the time of the flight j scheduled for the day. And let I = [I1, . . . , Ix, Ix+1, . . . , Ix+y] be the set of intervals for each flight scheduled for the day with x domestic and y international flights. Each interval represents a vertex of the graph G. For each vertex, we add the tolerance, ti, i = 1, . . . , v The minimum number of gates that must be used is equal to the chromatic number of the corresponding tolerance graph (each color corresponds to a gate). Corresponding computer program written in SageMath is given in figure 5.2. And its output is obtained is given in figure 5.3.

Departure-Cochin International Airport, 23 Aug 2021

Flight	City	Time	I or D
0	Doha	1.22	I
1	Sharjah	3.18	I
2	Kuwait	3.31	I
3	Sharjah	3.46	I
4	Doha	4.10	I
5	Dubai	4.30	I
6	Dubai	5.30	I
7	Doha	8.12	I
8	Muscat	8.23	I
9	Abu Dhabi	10.44	I
10	Male	9.27	I
11	Male	10.26	I
12	Dubai	10.17	I
13	Riyadh	11.28	I
14	Male	13.33	I
15	Male	13.48	I
16	Male	15.30	I
17	Dubai	16.02	I
18	Sharjah	16.10	I
19	Sharjah	16.23	I
20	Sharjah	20.03	I
21	Dubai	19.03	I
22	Sharjah	19.41	I
23	Delhi	21.00	D
24	Doha	21.15	I
25	Doha	23.32	I
26	Dubai	23.49	I
27	Hyderabad	5.15	D
28	Mumbai	6.13	D
29	Indore/Bangalo	6.40	D
30	Bangalore	7.33	D
31	Delhi	8.40	D
32	Kolkata	8.47	D
33	Bangalore	9.34	D
34	Delhi	9.46	D
35	Delhi	9.40	D
36	Mumbai	10.16	D
37	Mumbai	11.30	D
38	Agatti Island	11.43	D
39	Lucknow	12.27	D
40	Ahmedabad	12.45	D
41	Hyderabad	13.50	D
42	Bangalore	14.05	D
43	Chennai	14.43	D
44	Bangalore	15.31	D
45	Bangalore	16.26	D
46	Kolkata	17.15	D
47	Mumbai	17.20	D
48	Mumbai	17.26	D
49	Delhi	18.03	D
50	Chennai	18.12	D
51	Kannur	18.19	D
52	Hyderabad	19.26	D
53	Delhi	19.30	D
54	Delhi	19.30	D
55	Mumbai	19.54	D
56	Mumbai	20.01	D
57	Bangalore	20.40	D
58	Kolkata	21.16	D
59	Delhi	22.59	D
60	Delhi	23.26	D

Figure 5.1: table

From figure 5.3 we obtain the following informations: The minimum number of gates required for 61 flights is 11. Furthermore the additional information obtained is that gate 0 is used for flights 4,28,32,37 etc...and so on. . .

```
intoll=[(22.22,1.22,0.3),(12.18,3.18,0.3),(12.31,3.31,0.3),(12.46,3.46,0.3),(1.10,4.10,0.3),(1.3
0,4.30,0.3),(2.30,5.30,0.3),(5.12,8.12,0.3),(5.23,8.23,0.3),(7.44,10.44,0.3),(6.27,9.27,0.3),(7.
26,10.26,0.3),(7.17,10.17,0.3),(8.28,11.28,0.3),(10.33,13.33,0.3),(10.48,13.48,0.3),(12.30,15
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.2),(13.31,15.31,0.2),(14.26,16.26,0.2),(15.15,17.15,0.2),(15.20,17.20,0.2),(15.26,17.26,0.2),
(16.03,18.03,0.2),(16.12,18.12,0.2),(16.19,18.19,0.2),(17.26,19.26,0.2),(17.30,19.30,0.2),(17
.30,19.30,0.2),(17.54,19.54,0.2),(18.01,20.01,0.2),(18.40,20.40,0.2),(19.16,21.16,0.2),(20.59
,22.59,0.2),(21.26,23.26,0.2)]

def schedule_flights(intoll):

    g=graphs.ToleranceGraph(intoll)

    from sage.graphs.graph_coloring import chromatic_number

    cronum=chromatic_number(g)

    from sage.graphs.graph_coloring import all_graph_colorings

    GraphColoring=all_graph_colorings(g,cronum)

    return[g,cronum,next(GraphColoring)]

schedule_flights(intoll)
```

Figure 5.2: SageMath input

```
[Graph on 61 vertices,
 11,
 {0: [4, 28, 32, 37, 16, 48, 52, 58, 60, 0, 1, 2, 3],
  1: [5, 29, 33, 38, 17, 20, 25],
  2: [6, 7, 36, 39, 43, 45, 50, 23],
  3: [27, 34, 30, 40, 18, 49, 24],
  4: [35, 14, 19, 51, 56, 26],
  5: [8, 13, 41, 44, 46, 53, 59],
  6: [9, 15, 47, 54],
  7: [10, 42, 21],
  8: [11, 22],
  9: [12, 55],
  10: [31, 57]}]
```

Figure 5.3: SageMath output

Chapter 6

CONCLUSION

In various fields, graph theory has a diverse class of applications. It provides an approach to systematically testing the structure of given data and hence exploring connections between them by constructing graphs. In particular, Interval graphs along with its properties are used for wide variety of applications in many fields. Some of them are discussed here too.Interval graphs are an interesting case because biologists first developed them. Since Interval graphs are the generalization of Tolerance graphs, these applications are also valid to them. In this project, various applications of interval graphs are discussed and it gives a brief idea of the kind of real life situations in which interval graphs plays a major role. Several uses are proposed in the fight against the COVID-19 epidemic since graphs appear as a natural tool in shaping the various problems associated with this global epidemic by slowing down the process of transmitting the virus; grocery shop problem, room scheduling problem and flight scheduling problem are the examples.

REFERENCES

[1] Wikipedia

[2] An Introduction to Algorithmic Graph Theory: intro-agt-v1b-part1.pdf (haifa.ac.il)

[3] Interval graph word problem:
https://www.youtube.com/watch?v=xtimIa0QWggt=308s

[4] John R. Jungck and Rama Viswanathan, Graph Theory for Systems Biology: Interval Graphs, Motifs, and Pattern Recognition. In: Raina Robeva, editor, Algebraic and Discrete Mathematical Methods for Modern Biology

[5] Interval Graphs- 'The Case of the Missing Files'

[6] AMO - Advanced Modeling and Optimization, Volume 11, Number 3, 2009 Interval Tree and its Applications, Anita Pal and Madhumangal Pal

[7] Interval Graph- Berge Mystery https://slideplayer.com/slide/8044918/

[8] Martin Charles GOLUMBIC, Clyde L. MONMA , William T. TROTTER Jr , Tolerance Graphs, Discrete Applied Mathematics 9 (1984) 157-170 North-Holland

[9] Dean Crnković , Andrea Švob , Application of Tolerance Graph to combat Covid-19 pandemic

[10] The Sage Development Team, Sage Tutorial, Release 9.4

[11] SageMath- Introduction, Basic Arithmetic functions https://www.youtube.com/watch?v=2IOozavl21Q

[12] SageMath- Elementary number theory https://www.youtube.com/watch?v=22z6tRxjXqMt=848s

[13] Introduction to SageMath and Graph Theory https://www.youtube.com/watch?v=vBd22K-Nuj4t=675s

[14] **Tutorial- Programming in sagemath and python https://doc.sagemath.org/html/en/thematic**$_{t}$*utorials/tutorial − programming − python.html*