Project Report

On

A STUDY OF DEALING INFECTIOUS DISEASES MATHEMATICALLY

Submitted

in partial fulfilment of the requirements for the degree of

MASTER OF SCIENCE

in

MATHEMATICS

by

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CERTIFICATE

This is to certify that the dissertation entitled, **A STUDY OF DEALING INFECTIOUS DISEASES MATHEMATICALLY** is a bonafide record of the work done by Ms. **RANIYA J ANTONY** under my guidance as partial fulfillment of the award of the degree of **Master of Science in Mathematics** at St. Teresa's College (Autonomous), Ernakulam affiliated to Mahatma Gandhi University, Kottayam. No part of this work has been submitted for any other degree elsewhere.

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DECLARATION

I hereby declare that the work presented in this project is based on the original work done by me under the guidance of **SMT VEENA V.S**, Assistant Professor, Department of Mathematics, St. Teresa's College(Autonomous), Ernakulam and has not been included in any other project submitted previously for the award of any degree.

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Chapter 1

Introduction and History

1.1 INTRODUCTION

Mathematical models can project how infectious diseases progress to show the likely outcome of an epidemic and help inform public health interventions. Models use basic assumptions or collected statistics along with mathematics to find parameters for various infectious diseases and use those parameters to calculate the effects of different interventions, like mass vaccination programmes. The modeling can help decide which interventions to avoid and which to trail, or can predict future growth patterns, etc. COVID-19 is one of the major examples among infectious diseases. Six feet was proposed as a safe separation distance to prevent the spread of COVID-19 Diseases transmission can also occur through contact of contaminated surfaces, but this work focuses on transmission via inhalation of expelled aerosols during coughing/sneezing and breathing/talking and the impact of environmental factors.

1.2 HISTORY

The modelling of infectious diseases is a tool that has been used to study the mechanism by which diseases spread to predict the future course of an outbreak and to evaluate strategies to control an epidemic. The earlier account of mathematical modelling of the spread of diseases was carried out in 1760 by Daniel Bernoulli, trained as a physician; Bernoulli created a mathematical model to define the practice of

inoculating against smallpox. First model in mathematical epidemiology is the work of Daniel Bernoulli (1700-1782) on inoculation against smallpox. In the eighteenth century smallpox was an epidemic. G. R. Phaijoo and D. B. Gurung established that dengue is spreading in new areas due to people movement. They considered a multipath model to assess the influence of temperature and human movement on the transmission dynamics of dengue disease. Dynamics of vector and host populations are investigated with different human movement rates and different temperature levels. N. Pipatsart et al. discussed adaptive random network models to describe human behavioural change throughout epidemics and performed stochastic simulations of SIR epidemic models on adaptive random networks.

Chapter 2

Modelling of infectious diseases

The modeling of infectious diseases is a tool that has been used to study the mechanisms by which diseases spread, to predict the future course of an outbreak and to evaluate strategies to control an epidemic.

2.1 Steps of mathematical modelling

The stages involved in mathematical modelling are formulation, solution, interpretation and validation.Emphasizes that mathematical modeling is a non-linear process that includes five interrelated steps: (i) Identify and simplify the real- world problem situation. (ii) Build a mathematical model. (iii) Transform and solve the model. (iv) Interpret the model. (v) Validate and use the model.

2.2 Importance of Mathematical Modelling

Mathematical modelling is capable of saving lives, assisting in policy and decision-making, and optimizing economic growth. It can also be exploited to help understand the Universe and the conditions needed to sustain life,used to study the mechanisms by which diseases spread, to predict the future course of an outbreak and to evaluate strategies to control an epidemic.

Chapter 3

EPIDEMIC MODELLING

Mathematical modeling of infectious diseases has become a key tool in order to understand, predict and control the spread of infections.The aim of epidemic modelling is thus to model the spread of a disease in a population made up of a (possibly large) integer number of individuals. Population is divided into classes of susceptible, infective and recovered individuals.Disease dynamics can then be characterized by a mathematical description of each individual's transitions between compartments, subject to the state of the remaining individuals.

3.1 TYPES OF EPIDEMIC MODELS

3.1.1 Stochastic

"Stochastic" means being or having a random variable. A stochastic model is a tool for estimating probability distributions of potential outcomes by allowing for random variation in one or more inputs over time. A stochastic model, in its formulation, takes into account the random nature of an infectious disease. The stochastic model we study here is based on the "birth-and-death process with immigration" (BDI for short), which was proposed in the study of population growth or extinction of some biological species The stochastic mathematical models of infectious diseases represent a more realistic approach to epidemics, because they allow the recognition of the initial patterns in an epidemic the analysis of the spatial distribution of case numbers in a given location, and allow estimations about the duration of an epidemic

3.1.2 Deterministic

In deterministic models, the output of the model is fully determined by the parameter values and the initial conditions. When dealing with large populations, as in the case of tuberculosis, deterministic or compartmental mathematical models are often used. In a deterministic model, individuals in the population are assigned to different subgroups or compartments, each representing a specific stage of the epidemic. The transition rates from one class to another are mathematically expressed as derivatives, hence the model is formulated using differential equations. While building such models, it must be assumed that the population size in a compartment is differentiable with respect to time and that the epidemic process is deterministic. In other words, the changes in population of a compartment can be calculated using only the history that was used to develop the model

3.1.3 The SIR Model

W.O. Kermack and A.G Kendrick created a model in which they considered a fixed population with only three compartments: susceptible, S (t); infected, I (t); and recovered, R (t). S (t) is used to represent the individuals not yet infected with the disease at time t, or those susceptible to the disease of the population. I (t) denote the individuals of the population who have been infected with the disease and are capable of spreading the disease to those in the susceptible category. R (t) is the compartment used for the individuals of the population who have been infected and then removed from the disease, either due to immunization or due to death. Those in this category are not able to be infected again or to transmit the infection to others. The second set of dependent variables represents the fraction of the total population in each of the three categories. So, if N is the total population $s(t) = S(t)/N$, the susceptible fraction of the population, $i(t) = I(t)/N$, the infected fraction of the population, and $r(t) = R(t)/N$, The recovered fraction of the population.

3.1.4 Other Compartmental Models

There are many modifications of the SIR model, including those that include births and deaths, where upon recovery there is no immunity (SIS model), where immunity lasts only for a short period of time (SIRS), where there is a latent period of the disease where the person is not infectious (SEIS and SEIR), and where infants can be born with immunity (MSIR).

3.2 Continuous Time Modelling

Anderson and May (1991) Diekmann and Heesterbeek (2000) Keeling and Rohani (2008) Gave an introduction to epidemic modelling using primarily deterministic models based on ordinary differential equations (ODEs) in the setting of the susceptible-infective-recovered (SIR) model and its extensions. In a continuous model, events can take place at every point in time. For example, the time between birth and death can be any positive decimal number. Let S(t), I(t) and R(t) Denote the number at time t of susceptible, infective and recovered individuals, respectively.

$$\frac{ds(t)}{dt} = -\frac{\beta}{N} S(t)I(t) \tag{3.1}$$

$$\frac{dI(t)}{dt} = \frac{\beta}{N} S(t)I(t) - \gamma I(t) \tag{3.2}$$

$$\frac{dR(t)}{dt} = \gamma I(t) \tag{3.3}$$

Where the parameter $\beta \geq 0$ is the transmission rate and $\gamma \leq 0$ describes the removal rate. The initial condition is given by S (0), I (0), which are known integers, and R (0) = 0. In a population of fixed size N=S (0) + I (0) the expression for

$$\frac{dR(t)}{dt} \tag{3.4}$$

in the above ODE system is redundant because R (t) is implicitly given as

$$N - S(t) - I(t) \tag{3.5}$$

Chapter 4

Disease Transmission

Epidemics of infectious diseases among humans and other animals result from the transmission of a pathogen either directly between hosts or indirectly through the environment or intermediate hosts. The environment is important for the survival of intermediate hosts and vectors, which can affect the efficiency of transmission. Infectious diseases are often spread through direct contact.

4.1 How to prevent disease transmission

Because infectious diseases can spread through direct or indirect contact, everyone is at risk of illness. You have a higher risk of becoming ill when you're around sick people or in areas susceptible to germs. If you work in or visit a care centre, a day-care centre, a hospital, or a doctor's office, takes extra precautions to protect you.

4.1.1 Illness

Something as simple as touching a doorknob, elevator button, light switch, or another person's hand increases the likelihood of coming in contact with germs that can make you sick. The good news is that a few simple precautions can prevent some disease transmission. For example, make sure you wash your hands frequently and thoroughly. Use soap and warm water and vigorously rub your hands together for at least 20 seconds. If you can't wash your hands, use an alcohol-based hand sanitizer. Washing your hands is the gold standard though. Other tips to prevent the spread of disease in areas with germs include: • wash your hands or use hand sanitizer before handling food and after shaking hands • always wash with soap and water if your hands are visibly soiled • try to minimize touching your mouth or nose with your hands • avoid sick people, if possible • wear disposable gloves to avoid contact with blood and faces • use disposable gloves when caring for an ill person • cover your mouth when you sneeze and cough and wash your hands afterward • teach children not to put their hands or objects in their mouths • sanitize toys and changing tables

4.1.2 Foodborne illness

Dangerous organisms can thrive in improperly prepared food. Avoid cross-contamination by keeping raw meats and produce separate. Use different preparation surfaces for raw meats and wash surfaces and utensils thoroughly. Freeze or refrigerate perishable foods and leftovers promptly. According to the United States Department of Agriculture, you should set your refrigerator to 40°F (4°C) or below and your freezer to 0°F (-18°C) or below. Cook meats to a minimum internal temperature of 145°F (63°C). Cook ground meats to 160°F (71°C) and poultry to 165°F (73°C).

4.1.3 Insects and animals

When camping or enjoying wooded areas, wear long pants and long sleeves. Use insect repellent and mosquito netting. Don't touch animals in the wild. Don't touch sick or dead animals.

4.1.4 Vaccinations

Stay up to date on vaccinations, especially when traveling. Don't forget to keep your pet's vaccinations current, too. Vaccinations can drastically reduce your risk of becoming ill with some infectious diseases. If you can avoid a particular disease, you can also prevent the spread of the disease. There are different types of vaccinations, such as those to prevent: • measles • mumps • influenza • human papillomavirus

4.2 Infectiousness comprises three major components: biological, behavioural and environmental.

4.2.1 Biological infectiousness over time after infection for three different human pathogens

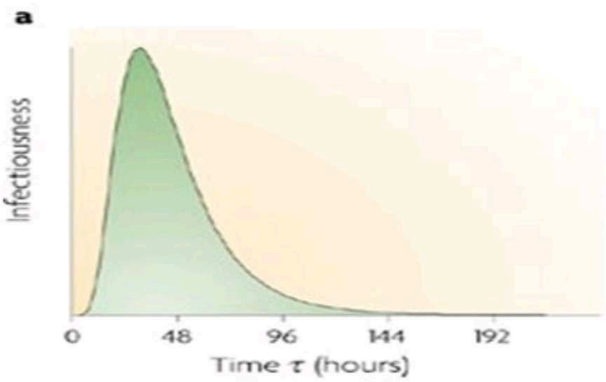

Figure 4.1: Influenza A:based on viral shedding in experimental human infections.

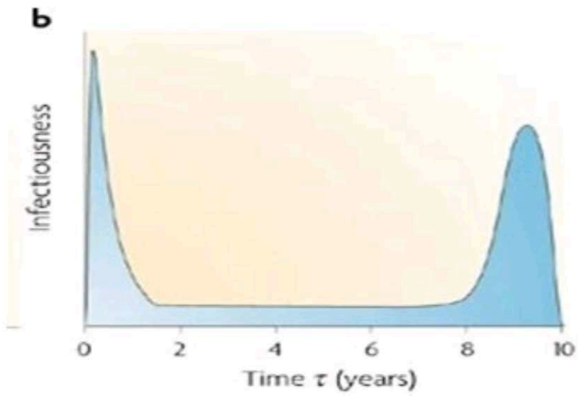

Figure 4.2: HIV-1 based on retrospective analysis of HIV-1 discordant couples and viral load data.

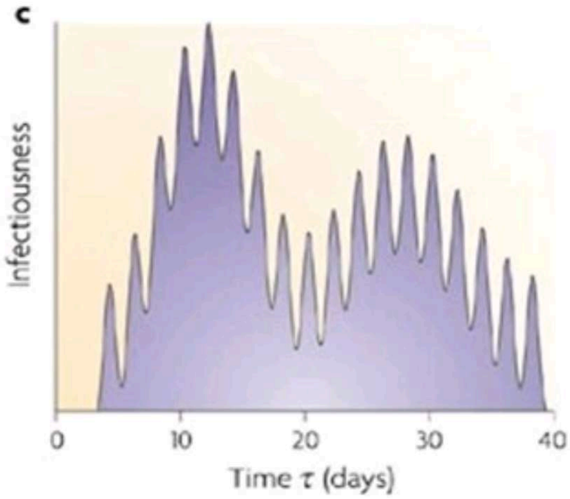

Figure 4.3: Malaraia: infectiousness to mosquitoes of infected humans based on the detection of infectious gametocytes in the blood after therapeutic treatment of syphilis by inoculation with plasmodium vivax.

4.2.2 Pathogen Transport and Transmission risk

An integrated modelling approach has been developed to better understand the relative impacts of different expiratory and environment factors on airborne pathogen transport and transmission, motivated by the recent COVID-19 pandemic. Computational fluid dynamics (CFD) modelling was used to simulate spatial-temporal aerosol concentrations and quantified risks of exposure as a function of separation distance, exposure duration, environmental conditions and face coverings.

4.3 Expiratory events and aerosol generation

4.3.1 Coughs and sneezes

Expiratory actions such as coughing and sneezing can yield thousands of minor respiratory droplets extending in size from $1 - 1000\mu$m that are expelled from the nose and mouth at high velocities. During a sneeze, respiratory droplets are expelled at velocities over 20m/s for brief periods typically lasting up to 0.25 s. Expiratory actions such as coughing and sneezing can yield thousands of minor respiratory droplets extending in size from $1 - 1000\mu$m that are expelled from the nose and mouth at high velocities. During a sneeze, respiratory droplets

are expelled at velocities over 20m/s for brief periods typically lasting up to 0.25 s. Expiratory actions such as coughing and sneezing can yield thousands of minor respiratory droplets extending in size from 1 – 1000μmthat are expelled from the nose and mouth at high velocities. During a sneeze, respiratory droplets are expelled at velocities over 20m/s for brief periods typically lasting up to 0.25 s. Previous studies have shown that expelled respiratory droplets can travel as far as 7 – 8 m (23-26 ft.). Higher temperatures increase the vapour pressure and volatilization of respiratory droplets, while higher relative humidity decreases the amount of volatilization. Their model was compared to experiments and showed good correlation. Performed computational fluid dynamics (CFD) simulations of the impact of wind and relative humidity on the transport and dynamics of respiratory droplets and confirmed that micro droplets follow the airflow streamlines well and can travel further than 6 feet.

4.3.2 Breathing and talking

Breathing and talking yield fewer and smaller droplets per exhalation than coughing or sneezing. Talking can yield several times more droplets than breathing, and singing can yield some times more droplets than talking. The exhaled velocity during breathing or talking is on the order of 1 m/s assuming a mouth opening of 4 cm2, an exhaled volume of 0.5 – 1 L, and a breathing rate of 16 breaths/s (3.75 s/breath). This yields a Reynolds number on the order of 1000, which is 30 – 40 times less than the Reynolds number for coughing or sneezing. Thus, the exhaled aerosols during breathing and talking have much lower momentum than coughing or sneezing and do not propagate as far. However, because the sizes of the droplets that are emitted during tidal breathing are small, the exhaled aerosol plume can remain suspended for long periods. Thus, despite the lower viral load per exhalation event relative to coughs or sneezes, the persistence of the minor aerosolized droplets and non-stop nature of breathing and/or talking can increase the potential for transmission, especially in enclosed spaces with low fresh-air exchange.

4.4 Graphical representation of the spread of infectious diseases

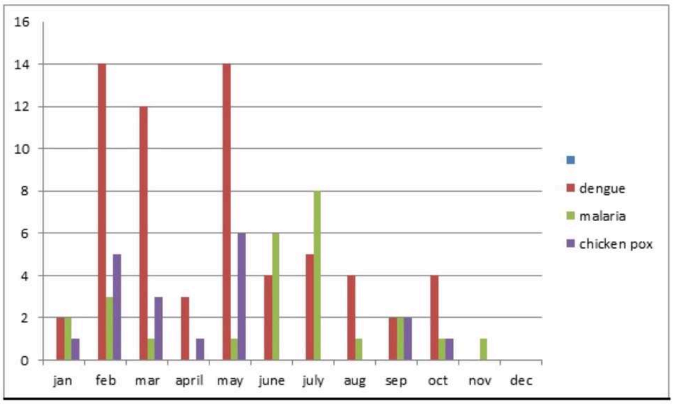

Figure 4.4: Figure in Latex

This is the graphical representation of spread of infectious diseases. And this data collected from Govt. Hospital angamaly .The graph showing the number of infected individuals from the disease malaria, dengue and chicken pox. And the horizontal axis showing the month and the vertical axis showing the number of infected.

4.4.1 Malaria

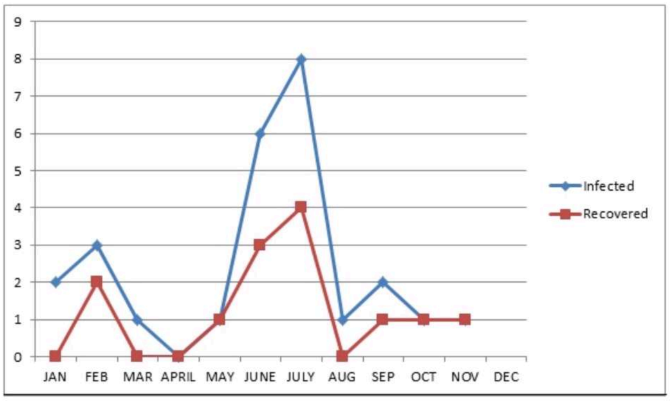

Figure 4.5: Figure in Latex

This is the graphical representation of the infectious disease malaria. The vertical axis showing the number of infected and recovered and

the horizontal axis showing the month. The graph showing the number of infected and recovered from the disease malaria. From the graph we can see that the number of infected is higher than the number of recovered.

4.4.2 Dengue

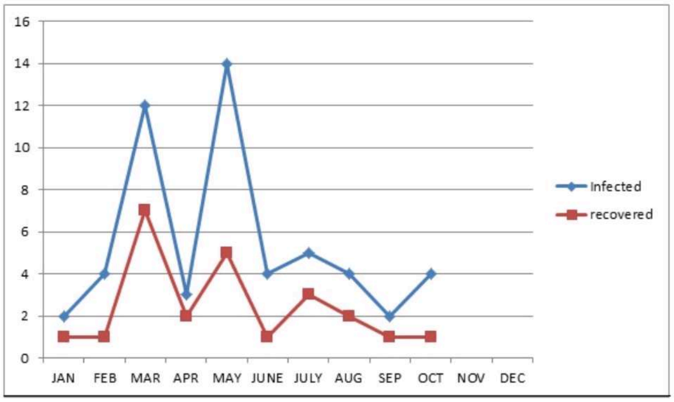

Figure 4.6: Figure in Latex

This is the graphical representation of the infectious disease dengue. The vertical axis showing the number of infected and recovered and the horizontal axis showing the month. The graph showing the number of infected and recovered from the disease dengue. From the graph we can see that the number of infected is higher than the number of recovered.

4.4.3 Chicken pox

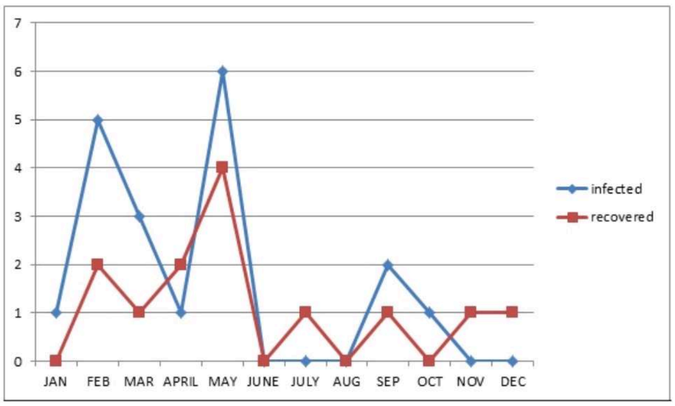

Figure 4.7: Figure in Latex

This is the graphical representation of the infectious disease Chicken pox. The vertical axis showing the number of infected and recovered and the horizontal axis showing the month. The graph showing the number of infected and recovered from the disease chicken pox. From the graph we can see that the number of infected is higher than the number of recovered.

From the study of above three graphs of the particular disease malaria, dengue and chicken pox, I arrive at a conclusion that the number of infected is always higher than the number of recovered. And the recovered is always less than the infected .This is the study that I have been made. And I arrive at a conclusion that the number of infected is higher than the recovered.

Chapter 5

Applications in Modelling of Infectious Diseases

Mathematical models are being increasingly used to understand the transmission of infections and to evaluate the potential impact of control programmes in reducing morbidity and mortality.Determining optimal control strategies against new or emergent infections, such as SARS-CoV-2, zika or Ebola, or against HIV, tuberculosis and malaria.Predicting the impact of vaccination strategies against common infections such as measles and rubella.Application of mathematical models to disease surveillance data can be used to address both scientific hypotheses and disease-control policy questions. Models exactly represent the real problem situations. Models help managers to take decisions faster and more accurately. They typically offer convenience and cost advantages over other means of obtaining the required information on reality.

Chapter 6

Conclusion

Mathematical modelling with spatial effects plays a significant role in characterizing and understanding the spread of particular infectious diseases. Epidemiologists use mathematical models in order to track the progress of most infectious diseases. They may also discover the likely outcome of an epidemic or to help manage them by vaccination.Mathematical models are a key tool for guiding public health measures, and outputs from epidemiological modelling analyses should be considered alongside numerous factors (such as potential economic and mental health effects of interventions) when deciding how to intervene.Models can demonstrate important principles about outbreaks and determine which interventions are most likely to reduce case numbers effectively.

REFERENCES

[1] Anderson, R. M. May, R. M. Infectious diseases of humans: dynamics and control (Oxford Univ. Press, 1991). – Google Scholar .

[2] Blower S, Bernoulli D (2004). "An attempt at a new analysis of the mortality caused by smallpox and of the advantages of inoculation to prevent it". Reviews in Medical Virology. 14 (5): 275–88. doi:10.1002/rmv.443.\protect\unhbox\voidb@x\protect\penalty\@M\PMID\protect\unhbox\voidb@x\protect\penalty\@M\15334536.\protect\unhbox\voidb@x\protect\penalty\@M\S2CID\protect\unhbox\voidb@x\protect\penalty\@M\8169180

[3] Daley DJ, Gani J (2005). Epidemic Modeling: An Introduction. New York: Cambridge University Press

[4] Mathematical Modeling and Control of Infectious Diseases retrieved from- https://www.hindawi.com/journals/cmmm/2017/7149154/

[5] Modelling and dynamics of infectious diseases, retrieved from https://1lib.in/book/688862/532b3f

[6] Modelling of infectious diseases- Wikipedia