NUMERICAL METHODS USING MATLAB

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IN MATHEMATICS

By

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CERTIFICATE

This is to certify that the dissertation entitled "NUMERICAL METHODS USING MATLAB" is a bonafide record of the work done by MARIA GOLDWIN under my guidance as partial fulfillment of the award of the degree of Master of Science in Mathematics at St. Teresa's College (Autonomous), Ernakulam affiliated to Mahatma Gandhi University, Kottayam. No part of this work has been submitted for any other degree elsewhere.

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I hereby declare that the work presented in this project is based on the original work done by me under the guidance of Miss. Alka Benny, Assistant Professor, Department of Mathematics, St Teresa's College(Autonomous), Ernakulam and has not been included in any other project submitted previously for the award of any degree.

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Chapter 1

INTRODUCTION

1.1 Numerical Analysis:

Numerical Analysis is a branch of mathematics which deals with the study of approximation techniques for solving problems by taking into account the possible errors, which may occur due to our approximation. The overall goal of this field is the design and analysis of techniques to give approximate but accurate solutions to hard problems and for extracting useful information from available solutions.

1.2 Numerical methods:

Numerical method is a set of procedures done to find the solution of a problem together with the error estimates. This enables is to find the solution of complex problems with numerous steps but using simple operations. Algorithms, computational steps or flow charts are provided for some of the numerical methods and these can easily be transformed into a computer program by including suitable input/output statements. It is well known that computational methods are important tool for numerical methods. Till 1950, numerical methods could only be implemented by manual computations, but the rapid technological advances resulted in the production of computing machines which are faster, economical and smaller in size.

1.3 Why do we use Numerical Methods???

One of the main reason for introducing numerical methods is that it is not always possible to find as analytical solution of a problem as it may be really time consuming. Another reason is that many complex real world problems cannot be solved using existing analytical methods. Also it is not possible to solve highly nonlinear equations using analytical techniques. In all these cases we can apply numerical methods to find an approximate solution for our problem.

Actually analytical and numerical methods serve different purposes. Analytical methods always helps us to understand the mechanism and physical effects through the model problem whereas numerical methods helps us to solve complex problems in real life physically or geometrically.

1.4 Outline:

In this project we deal with numerical methods used in finding solutions of differential equations. Many problems in science and engineering can be reduced to the problem of solving differential equations satisfying certain given conditions. To describe various numerical methods for the solution of O.D.E. we consider the general first order differential equation

$$\frac{dy}{dx} = f(x, y)$$

With initial condition $y(x_0) = y_0$ The methods so developed can be applied to the solution of system of first order equations and yields the solution of system as either, a set if tabulated values of x and y or a series for y in terms of power of x, form which the values of y can be obtained by direct substitution.

We discuss some basic methods like Taylor's Series and Picard's method and introduce the methods such as Euler's method, Modified Euler's method and R-K method to solve the I.V.P. In these methods we evaluate the next point on the curve in short steps by performing iteration until required accuracy is achieved. These methods are called step by step methods. We also introduce predictor method -Milne method to solve I.V.P's which is a multi-step method.And most importantly we have used the software Matlab .We have suitably programmed programs for different numerical methods mentioned earlier and with its graphical support compared the solution got by using the numerical methods and actual solution.We also introduce the basic concepts of perturbation equations using numerical methods and Matlab and solved

Airy's equations using power series method and plot its graph with the help of Matlab.

1.5 MatLab

It is well known that the programming effort is considerably reduced by using standard functions and subroutines. Several software for numerical method are available and are being rapidly used by engineering students. One such software is MATLAB (Matrices Laboratory). It was developed by Prof. Cleve Moler.

Using MATLAB it is possible to implement most of the numerical methods. There are other software such that FORTRAN, C, C++, etc. for implementing numerical methods, but MATLAB is widely using because it contains a large no. of functions that occurs proven numerical libraries, such as LINPACK and EISPACK. This means that many common task for solution of simulations equations can be accomplished with a single function call. Graphic support of the computational result is another merit of MATLAB. All numerical objects are treated as double precision arrays thus there is no need to declare datatype and carryout conversions. So we made a good attempt to use MATLAB for our analysis.

Chapter 2

PRELIMINARIES

2.1 Finite Difference

The calculus of finite differences deals with the changes that take place in the values of the function (dependent variable), due to the finite changes in the independent variable. Suppose that the function $y = f(x)$ is tabulated for the equally spaced values $x = x_0, x_0+h, x_0+2h, ...x_0+nh$ giving $y = y_0, y_1, ...y_n$. To determine the values of $f(x), f^{'}(x), f^{''}(x)$ for some intermediate values of$x$, the following 3 types of differences are useful. The calculus of finite differences form is the basis of many process and it is used in the numerical analysis.

2.1.1 Forward Difference

Let $y_0, y, ...y_n$be the values of the function $y = f(x)$, at equally spaced $x_0, x_1, ...x_n$ respectively.

Let $x_1 - x_0 = x_2 - x_1 = ... = x_n - x_{n-1} = h$

Then the differences are

$y_1 - y_0, y_2 - y_1, ...y_n - y_{n-1}$ then we denote $\Delta y_0 = y_1 - y_0, \Delta y_1 = y_2 - y_1 ....$ and therefore we have$\Delta y_{n-1} = y_n - y_{n-1}$

And these are called First order forward differences or First differences. Here the notation Δ is called the forward difference operator. The difference of first order difference are called Second order differences.

$$\text{They are } \Delta^2 y_0 = \Delta y_1 - \Delta y_0$$

$$\Delta^2 y_1 = \Delta y_2 - \Delta y_1$$

$$\Delta^2 y_{n-1} = \Delta y_n - \Delta y_{n-1}$$

Similarly we can find upto n'th order differences using below equation.

$$\Delta^n y_i = \Delta^{n-1} y_{i+1} - \Delta^{n-1} y_i \ ; i = 1, 2, 3 \dots n$$

$$\text{If} \quad y_i = f(x_i), \text{ then} \Delta y_i = y_{i+1} - y_i$$

$$\Delta f(x_i) = f(x_i + h) - f(x_i)$$

There fore

$$\Delta^2 y_0 = \Delta y_1 - \Delta y_0$$

$$= y^2 - 2y_1 + y_0$$

2.1.2 Backward Difference

The difference $y_1 - y_0, y_2 - y_1, ...y_n - y_{n-1}$are denoted by$\nabla y$=$\nabla y_0$,$\nabla y_1, ...$ ∇y_n, called the first order backward differences and the operator∇ is called backward difference operator.

The equations are showed below.

$$\nabla^2 y_2 = \nabla y_2 - \nabla y_1$$

$$\nabla^2 y_3 = \nabla y_3 - \nabla y_2$$

$$\nabla^2 y_n = \nabla y_n - \nabla y_{n-1},$$

called second order backward differences.

Similarly,

$$\nabla^3 y_i = \nabla^2 y_i - \nabla^2 y_{i-1}$$

$$\nabla^n y_i = \nabla^{n-1} y_i - \nabla^{n-1} y_{i-1}$$

2.2 Interpolation

Suppose we are given the following values of y=f(x) for a set of values of x.

x	x_0	x_1	x_2	. . .	x_n
y	y_0	y_1	y_2	. . .	y_n

Then the process of finding the values if y corresponding to any values of x=x_i between x_0 and x_n is called interpolation i.e It is the technique of estimating the values of a function for any intermediate value if the independent variable. If the function f(x) is known explicitly, then values of y corresponding to any values of x can easily be found. Conversely if the form of f(x) is not known (most cases it is so)it is very difficult to determine the exact form of f(x) with tabulated values (x_i, y_i). In such cases f(x) is replaced by a simple function $\phi(x)$ which assumes the same values as those of f(x) at the tabulated set of points. Any other values may be calculated from $\phi(x)$ which is known as the interpolating function or smoothing function. If $\phi(x)$ is a polynomial, then it is called the interpolating polynomial and the process is called polynomial interpolation.

2.3 Newton's Forward Interpolation Formula

Let the function y=f(x) take the values $y_0, y_1, y_2, ..., y_n$ at equally spaced arguments $x_0, x_1, ..., x_n$.Let h be the interval so that $x_r = x_0 + rh$.Let $\phi(x)$ be the interpolating polynomial of n^{th} degree which may be written in the form

$$\phi(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1) + a_3(x - x_0)(x - x_1)(x - x_2) + ...+$$

$$a_n(x - x_0)(x - x_1)...(x - x_{n-1}) \qquad (1)$$

where $a_0, a_1, a_2, ..., a_n$are constants to be determined such that

$$\phi(x_0) = y_0, \phi(x_1) = y_1, ..., \phi(x_n), = y_n$$

Putting $x = x_0, x_1, x_2, ..., x_n$successively in (1) we get

$$\phi(x_0) = a_0 \Rightarrow a_0 = y_0.$$

$$\phi(x_1) = a_0 + a_1(x_1 - x_0)$$

$$y_1 = a_0 + a_1 + h$$

$$\Rightarrow a_1 = \frac{y_1 - y_0}{h} = \frac{\triangle\, y_0}{h}$$

$$\phi(x_2) = a_0 + a_1(x_2 - x_0) + a_2(x_2 - x_0)(x_2 - x_1)$$

$$y_2 = y_0 + \frac{y_1 - y_0}{h}(2h) + a_2.2h.h$$

$$2h^2 a_2 = y_2 - 2y_1 + y_0 = \triangle^2 y_0$$

$$\therefore a_2 = \frac{\triangle^2 y_0}{2h^2} = \frac{\triangle^2 y_0}{2!h^2}$$

$$Similarly, a_3 = \frac{\triangle^3 y_0}{3!h^3}, ..., a_n = \frac{\triangle^n y_0}{n!h^n}$$

$$\therefore \phi(x) = f(x) = y_0 + \frac{\triangle y_0}{1!h}(x - x_0) + \frac{\triangle^2 y_0}{2!h^2}(x - x_0)(x - x_1) +$$

$$\frac{\triangle^3 y_0}{3!h^3}(x - x_0)(x - x_1)(x - x_2) + ... + \frac{\triangle^n y_0}{n!h^n}(x - x_0)...(x - x_{n-1})$$

Now let$x = x_0 + hu$

$$x - x_1 = x_0 + hu - (x_0 + h) = h(u - 1)$$

$$x - x_2 = x_0 + hu - (x_0 + 2h) = h(u - 2)$$

$\vdots$

$$x - x_{n-1} = h(u - (n - 1))$$

$$f(x) = f(x_0 + hu)$$

$$= y_0 + \frac{\triangle\ y_0}{1!h} hu + \frac{\triangle^2\ y_0}{2!h^2} hu.h(u - 1) + \frac{\triangle^3\ y_0}{3!h^3} hu.h(u - 1).h(u - 2)$$

$$+ ... + \frac{\triangle^n\ y_0}{n!h^n} hu.h(u - 1)...h(u - (n - 1))$$

$$\therefore y = y_0 + \frac{u}{1!} \triangle\ y_0 + \frac{u(u - 1)}{2!} \triangle^2\ y_0 + \frac{u(u - 1)(u - 2)}{3!} \triangle^3\ y_0$$

$$+ ... + \frac{u(u-1)...(u-(n-1))}{n!} \bigtriangleup^n y_0$$

This is called Newton's Forward Interpolation Formula.

Chapter 3

BASIC METHODS

Differential equations are among the most important mathematical tools used in producing models of Physical and Biological sciences and Engineering. We consider numerical methods for solving ordinary equations, i.e. those differential equations that have only one independent variable. We shall consider some of the methods commonly used to solve differential equations.

3.1 Taylor's Series Method

Consider the first order differential equation
$\frac{dy}{dx} = f(x, y)$with initial condition $y(x_0) = y_0$.
We can expand y(x) as a power series in$(x - x_0)$ in the nbd of x_0 by Taylor's series.

$$y(x) = y(x_0) + y'(x_0)(x - x_0) + \frac{y''(x_0)}{2!}(x - x_0)^2 + ...$$

put h=$x - x_0, x = x_0 + h$

$$y(x) = y(x_0) + hy'(x_0) + \frac{h^2}{2!}y''(x_0) + ...$$

$$y(x) = y_0 + hy'_0 + \frac{h^2}{2!}y''_0 + \frac{h^3}{3!}y'''_0 + ...$$

If $x = x_1 = x_0 + h$ then

$$y(x_1) = y_1 = y_0 + hy'_0 + \frac{h^2}{2!}y''_0 + ...$$

Now y can be expanded as Taylor's series about $x = x_1$ and we have

$$y(x_1 + h) = y(x_2) = y_2 = y_1 + \frac{h}{1!}y'_1 + \frac{h^2}{2!}y''_1 + ...$$

Continuing this way we find

$$y_{r+1} = y_r + \frac{h}{1!}y'_r + \frac{h^2}{2!}y''_r + ...$$

where $y_{r+1} = y(x_{r+1}) \qquad r = 1, 2, 3, ...$

The solution y(x) is given as a sequence $y_0, y_1, y_2, ...$

3.1.1 Example

Suppose $\frac{dy}{dx} = x^2y - 1, y(0) = 1$

Solution: Given $y' = x^2y - 1$

$x_0 = 0, y_0 = 1, h = 0.1$

Taylor's series is

$$y_{r+1} = y_r + \frac{h}{1!}y'_r + \frac{h^2}{2!}y''_r + ... \qquad (1)$$

$$y' = x^2y - 1$$

$$y'' = x^2y' + 2xy$$

$$y''' = x^2y'' + 2xy' + 2xy + 2y$$

$at(x_0, y_0) = (0, 1)$

$$y'_0 = -1, y''_0 = 0, y'''_0 = 2$$

Substituting r=0 in (1) we get

$$y_1 = y_0 + hy_0' + \frac{h^2}{2!}y_0'' + ...$$

$$y_1 = y(0 + 0.1) = 1 + 0.1(-1) + 0 + \frac{0.1^3}{3!}2 + ...$$

=0.9003

Hence the values of y at the point x=0.1 is given by 0.9003.

Also at $(x_1, y_1) = (0.1, 0.9003)$

put r=1 in (1) we get,

$$y_2 = y_1 + \frac{h}{1!}y_1' + \frac{h^2}{2!}y_1'' + ...$$

$y_1' = -0.990, \quad y_1'' = 0.170, \quad y_1''' = 1.784$

$y_2 = 0.9003 + 0.1(-0.990) + \frac{0.1^2}{2!}(0.170) + \frac{0.1^3}{3!}(1.784) + ...$

$y_2 = 0.8024$

3.2 Picard's Method

Consider the differential equation $\frac{dy}{dx} = f(x, y)$ (1)

It is required to find that particular solution of(1) which assumes the value y_0 when $x = x_0$.Integrating (1) between the limits

$$\int_{x}^{y} dy = \int_{x_0}^{x} f(x,y)dx$$

$$y = y_0 + \int_{x_0}^{x} f(x,y)dx \qquad (2)$$

As a first approximation y_1 to the solution,we put $y = y_0$ in f(x,y) and integrate (2),giving

$$y_1 = y_0 + \int_{x_0}^{x} f(x,y_0)dx$$

For the second approximation y_2,we put $y = y_1$ in f(x,y) and integrate (2),giving

$y_2 = y_0 + \int_{x_0}^{x} f(x,y_1)dx$

Continuing this process,we obtain $y_3, y_4, y_5, ...where$

$$y_n = y_0 + \int_{x_0}^{x} f(x,y_{n-1})$$

3.2.1 Example

$\frac{dy}{dx} = 3x + y^2$ with initial condition y=1 when x=0.

f(x,y)=$3x + y^2$ Picard's formula is $y = y_0 + \int_{x_0}^{x} f(x,y)dx$

$y = 1 + \int_{0}^{x} f(x,y)dx$

The first approximation is

$$y_1 = 1 + \int_0^x f(x, y_0)dx$$

$$= 1 + \int_0^x (3x + y_0^2)dx$$

$$= 1 + \int_0^x (3x + 1)dx$$

$$= 1 + [3\frac{x^2}{2} + x] - 0$$

$$= 1 + x + 3\frac{x^2}{2}$$

The second approximation is

$$y_2 = y_0 + \int_{x_0}^x f(x, y_1)dx$$

$$= 1 + \int_0^x [3x + y_1^2]dx$$

$$= 1 + \int_0^x [3x + (1 + x + 3\frac{x^2}{2})^2]dx$$

$$= 1 + \int_0^x [3x + 1 + x^2 + \frac{9}{4}x^4 + 2x + 3x^2 + 3x^3]dx$$

$$= 1 + \int_0^x [1 + 5x + 4x^2 + 3x^3 + \frac{9}{4}x^4]dx$$

$$= 1 + [x + 5\frac{x^2}{2} + 4\frac{x^3}{3} + 3\frac{x^4}{4} + \frac{9}{4}\frac{x^5}{5}]_0^x$$

$$y_2 = 1 + x + 5\frac{x^2}{2} + 4\frac{x^3}{3} + 3\frac{x^4}{4} + \frac{9}{20}$$

The third approximation involves squares of y_2 which is a big expression..So we stop with y_2.

when x=0.1,$y_2 = 1 + 0.1 + 5\frac{0.2^2}{2} + 4\frac{0.1^3}{3} + 3\frac{0.1^4}{4} + 9\frac{0.1^5}{20}$

=1.1264

when x=0.1 ,y=1.1264

when x=0.2, we get y=1.3120

Chapter 4

COMPARISON OF SOLUTIONS OF O.D.E's USING MATLAB

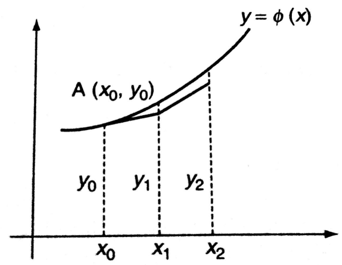
4.1 Euler's Method

Euler's method is one of the simplest and oldest method of finding numerical solution for differential equation. It is a step by step method because the values of y once computed by short steps ahead for equal intervals of the independent variable.

Consider the differential equation

$$\frac{dy}{dx} = f(x, y),\ y(x_0) = y_0$$

Let x_0, x_1,.... be equidistant values of x.Where $x_1 - x_0 = h = x_2 - x_1 =$ ..... So that $x_1 = x_0 + h, x_2 = x_0 + 2h$.... In Euler's method we approximate a curve in a small interval by a straight line.



Let $y = \phi(x)$ be the curve representing the actual solution. Equation of the tangent at (x_0, y_0) is ,

$$y - y_0 = [\frac{dy}{dx}]_{(x_0,y_0)}(x - x_0)$$

$$= f(x_0, y_0)(x - x_0)$$

$$y = y_0 + f(x_0, y_0)(x - x_0)$$

Since the curve in the interval $(x_0, x_0 + h)$ is approximated by this straight line, the value of y at $x = x_1$ is approximately,

$$y_1 = y_0 + f(x_0, y_0)(x_1 - x_0)$$

$$= y_0 + hf(x_0, y_0).$$

Similarly the curve in the interval (x_1, x_2) is approximated by the line through (x_1, y_2) and having slope (x_1, y_1)

$$y_2 = y_1 + hf(x_1, y_1)$$

Proceeding in this way we get the general formula,

$$y_{n+1} = y_n + hf(x_n, y_n), n = 0, 1, 2, 3.....$$

This is called Euler's algorithm.

4.2 Euler's Method-MATLAB

```
%eulers method
clear all
clc
a=0;
b=1;
y(1)=1;
n=10;
t(1)=a;
h=(b-a)/n;
f=@(y,t)(t+y);
for i=1:(n+1)
y(i+1)=y(i)+h*f(y(i),t(i));
t(i+1)=a+i*h;
fprintf('when t(i+1)=%f y(i+1)=%f\n',t(i),y(i))
end
plot(t,y,'-r','LineWidth',4)
```

hold on

*plot(t,-t-1+2*exp(t),'-b','LineWidth',4)*

hold off

4.2.1 Example:

Consider the differential equation

$$\frac{dy}{dx} = x + y \text{ and } \; y_0 = 1$$

We will find the approximate value of y corresponding to the point $x = 1$. For this we take $n = 10$ and $h = 0.1$.The various calculations are showed in table 1.

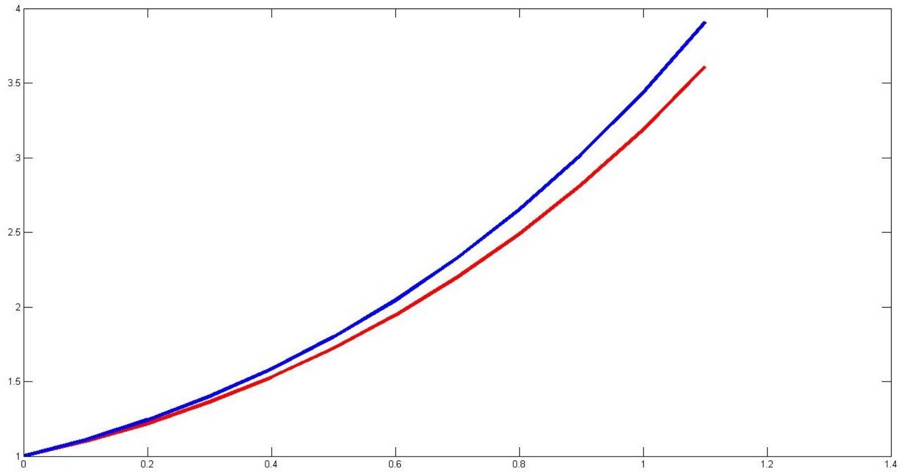
Table:1

x	y	$x + y = \frac{dy}{dx}$	New $y = Old\, y + 0.1 \times \frac{dy}{dx}$
0.0	1.00	1.00	$1.00 + 0.1 \times (1.00) = 1.10$
0.1	1.10	1.20	$1.10 + 0.1 \times (1.20) = 1.22$
0.2	1.22	1.42	$1.22 + 0.1 \times (1.42) = 1.36$
0.3	1.36	1.66	$1.36 + 0.1 \times (1.66) = 1.53$
0.4	1.53	1.93	$1.53 + 0.1 \times (1.93) = 1.72$
0.5	1.72	2.22	$1.72 + 0.1 \times (2.22) = 1.94$
0.6	1.94	2.54	$1.94 + 0.1 \times (2.54) = 2.19$
0.7	2.19	2.89	$2.19 + 0.1 \times (2.89) = 2.48$
0.8	2.48	3.29	$2.48 + 0.1 \times (3.29) = 2.81$
0.9	2.81	3.71	$2.81 + 0.1 \times (3.71) = 3.18$
1.00	3.18		

Thus the approximate values of y at $x = 1$ is 3.18.The actual solution of this differential equation is $y = -x - 1 + 2\exp^{x}$. The comparison of actual method and Euler's method results are shown in table 2.

Table:2

x	y (actual method)	y (Euler's method)
0.0	1.00	1.00
0.1	1.11	1.10
0.2	1.24	1.22
0.3	1.39	1.36
0.4	1.58	1.53
0.5	1.79	1.72
0.6	2.04	1.94
0.7	2.32	2.19
0.8	2.65	2.48
0.9	3.01	2.81
1.00	3.43	3.18



The blue one represents the graph of original solution and red one represents the graph of Euler's method solution. As we increase the values of n, the accuracy of the solution will increase. So we can conclude that Euler's method for solving ordinary differential equations.

4.3 Modified Euler Method

Even though Euler's method is simple and accurate method for finding the solution of O.D.E, time consuming is an issue of this method. Hence there is a modified Euler's method, which can be replace Euler's method in terms of accuracy as well as time consumption.

By Euler's method we have

$$y_1 = y_0 + hf(x_0 + y_0)$$

Now we can replace the equation by Modified Euler's method like,

$$y_1^1 = y_0 + \tfrac{h}{2} f(x_0, y_0) + f(x_0 + h), y_1)$$
$$y_1^2 = y_0 + \tfrac{h}{2} f(x_0, y_0) + f(x_0 + h), y_1^1)$$

We repeat this step still two consecutive values of y agree.

Proceeding like this we can find $y_2, y_3, ...$

4.4 Modified Euler's Method-MATLAB

```
clc
clear all
% initial condition
f=inline('x+y');
n = input('Enter the value of n:');
xn=input('enter the value of xn:');
x(1)=0;
y(1)=1;
h=input('enter the value of h:');
for i=1:n
y(i+1) = y(i)+h*f(x(i)+0.5*h,y(i)+0.5*h*f(x(i),y(i)));
x(i+1) = x(i)+h;
fprintf('when x(i+1)=%f y(i+1)=%6.4f\n',x(i),y(i))
end
plot(x,-x-1+2*exp(x),'-m','LineWidth',4)
hold on
plot(x,y,'+b','LineWidth',7)
hold off
```

4.4.1 Example:

Consider the equation,

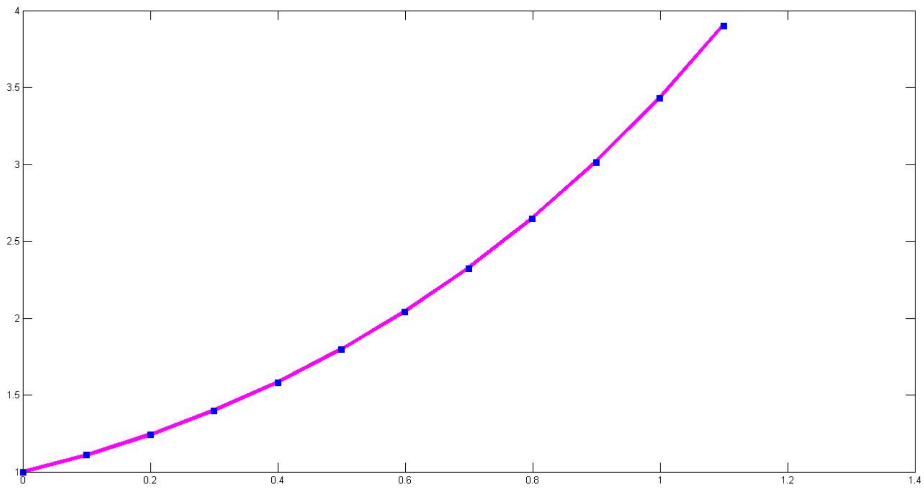
$$\frac{dy}{dx} = f(x, y) \text{ with initial condition } y(x_0) = y_0$$

The values of$y$ calculated by Modified Euler's method at the point $0.1, 0.2, 0.3$ and the values got by actual method are shown in table 3.

Table 3

x	y(actual value)	y(Modified Euler's method)
0	1.0000	1.0000
0.1	1.1105	1.1100
0.2	1.2428	1.2421
0.3	1.3997	1.3985
0.4	1.5836	1.5818
0.5	1.7974	1.7949
0.6	2.0448	2.0409
0.7	2.3275	2.3231
0.8	2.6510	2.6456
0.9	3.0192	3.0124
1.0	3.4365	3.4282

The magenta line represents the original solution and the blue dots represent the solution got by Modified Euler's method . So we can conclude that Modified Euler's can gives more accuracy as well as time consumption over Euler's method.



4.5 Runga-Kutta METHOD (R- K Method)

The Taylor's series method of solving differential equations numerically is restricted by the labour involved in finding the higher order derivatives. Two famous German mathematicians namely Runga and Kutta developed an algorithms named by R-K method to solve a differential equation in another way. The advantage of this method is that it requires only values of the function at some specified points. These methods agree with Taylor's series solution up to the terms in h r, where r is called order of the method and its value differs from method to method.

4.5.1 First order R-K Method

By Euler's method we have $y_1 = y_0 + \frac{h}{2}f(x_0, y_0) = y_0 + hy_0^{'}$ Where $y_0^{'} = f(x_0, y_0)$ Expanding by Taylor's series, $y_1 = y(x_0 + h) = y_0 + hy_0 + \frac{h^2}{2}y_0^{''} + ...$ It follows that the Euler's method agrees with the Taylor's solution up to the term in h. Hence Euler's method is the first order term in R-K method.

4.5.2 Second order R-K Method

The modified Euler's method gives

$$y_1 = y_0 + \frac{h}{2}[f(x_0, y_0 + f(x_0 + h, y_1)] \quad (1)$$

Substituting $y_1 = y_0 + hf(x_0, y_0)$ on the R.H.S of (1), we obtain

$$y_1 = y_0 + \frac{h}{2}[f_0 + f(x_0 + h, y_0 + hf(0) \;\; where f(0) = f(x_0, y_0) \quad (2)$$

Expanding L.H.S by Taylor's series, we get

$$y_1 = y(x_0 + h) = y_0 + hy_0^{'} + \frac{h^2}{2!}y_0^{'} + \frac{h^3}{3!}y_0^{'''} + \ldots \quad (3)$$

Expanding $f(x_0 + h, y_0 + hf_0)$ by Taylor's series for a function of two variables,(2)gives

$$y_1 = y_0 + \frac{1}{2}[hf_0 + hf_0 + h^2[(\frac{\partial f}{\partial x})_0 + (\frac{\partial f}{\partial x})_0] + O(h^3)]$$

$$= y_0 + hf_0 + \frac{h^2}{2}f_0' + O(h^3)$$

$$= y_0 + hy_0' + \frac{h^2}{2}y_0'' + O(h^3)$$

Comparing Eq(3)and Eq(4) it follows that the modified Euler's agrees with Taylor's series solution up to the term in h^2. Hence the modified Euler's method is the R-K method of second order.And the equation should be,

$$y_1 = y_0 + \frac{1}{2}[k_1 + k_2], \text{ where } k_1 = hf(x_0, y_0) \qquad , k_2 = hf(x_0 + h, y_0 + k_1)$$

4.5.3 Third order R-K Method

The third order R-K formula is

$$y_1 = y_0 + 1/6[k_1 + 4k_2 + k_3] \text{ Where, } \quad K_1 = hf(x_0, y_0)$$

$$k_2 = hf(x_0 + \tfrac{1}{2}h, y_0 + \tfrac{1}{2}k_1)$$

$$k_3 = hf(x_0 + h, y_0 + k'); \;\; k' = hf(x_0 + h, y_0 + k_1)$$

4.5.4 Fourth order R-K Method

The fourth order R-K method is most widely used and is popular and so it is referred to us R-K method.

$$y_1 = y_0 + 1/6[k_1 + 2k_2 + 2k_3 + k_4]$$

$$\text{Where} \quad k_1 = hf(x_0, y_0)$$

$$k_2 = hf(x_0 + \tfrac{1}{2}h, y_0 + \tfrac{1}{2}k_1)$$

$$k_3 = hf(x_0 + \tfrac{1}{2}h, y_0 + \tfrac{1}{2}k_2)$$

$$k_4 = hf(x_0 + h, y_0 + k_3)$$

4.6 R K Method-MATLAB

```
clc
clear all
% initial condition
f=inline('x+y');
x(1) = 0;
y(1) = 1;
h=input('enter the value of h:');
xn=input('enter the value of xn:');
n=input('enter the value of n:')
for i = 1:n
k1=h*f(x(i),y(i));
k2=h*f(x(i)+h/2,y(i)+k1/2);
k3=h*f(x(i)+h/2,y(i)+k2/2);
k4=h*f(x(i)+h,y(i)+k3);
y(i+1) = y(i)+1/6*(k1+2*(k2+k3)+k4);
x(i+1) = x(i)+h;
fprintf('when x(i+1)=%f y(i+1)=%6.4f\n',x(i),y(i))
end
plot(x,-x-1+2*exp(x),'-r','LineWidth',3)
hold on
plot(x,y,'+-b','LineWidth',5)
hold off
```

4.6.1 Example:

Consider the equation,

$$\frac{dy}{dx} = x + y; \quad y(0) = 1$$

$$K_1 = hf(x_0, y_0) = 0.2 \times 1 = 0.2000$$

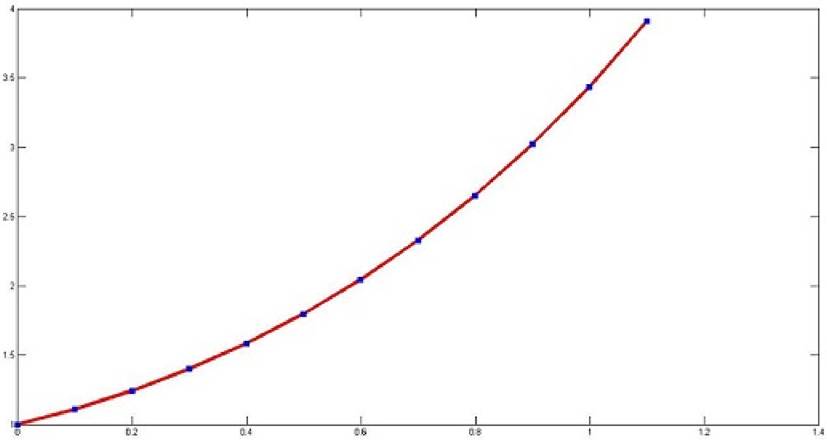
$$k_2 = hf(x_0 + \tfrac{1}{2}h, y_0 + \tfrac{1}{2}k_1) = 0.2400$$

$$k_3 = hf(x_0 + \tfrac{1}{2}h, y_0 + \tfrac{1}{2}k_2) = 0.2440$$

$$k_4 = hf(x_0 + h, y_0 + k_3) = 0.2888$$

$$y_1 = y_0 + \tfrac{1}{6}[k_1 + 2k_2 + 2k_3] + k_4] = \tfrac{1}{6} \times 1.4568 = 0.2428$$

x	y(actual value)	y(R K method)
0.1	1.1105	1.1103
0.2	1.2428	1.2428
0.3	1.399	1.3997
0.4	1.5836	1.5835
0.5	1.7974	1.7969



The red line represents the original solution and the blue dots represent the solution got by R K method . One of the merit of this method is that the operation is identical whether the differential equation is linear or nonlinear. R-K method has been constructed so as to give better accuracy and it requires only the function values at some selected points on the sub interval. Thus we can concludes that R-K method have better performance over other methods.

Chapter 5

PREDICTOR CORRECTOR METHOD

Consider the differential equation $\frac{dy}{dx} = f(x, y)$with $y(x_0) = y_0$.

Divide the range of x into a no. of subintervals of equal width h.

If x_i and x_{i+1} are consecutive points then $x_{i+1} = x_i + h$.

By Euler's method we have $y_{i+1} = y_i + hf(x_i, y_i), i = 1, 2, 3, ...$ (1)

By Modified Euler's method we have, $y_{i+1}\frac{h}{2}[f(x_i, y_i) + f(x_{i+1}, y_{i+1})]$ (2)

The value of $y_{i+1}$is first estimated by (1)and this value is substituted in the RHS of (2). Then we get a better approximation of y_{i+1} from (2). This value is again substituted in(2) to find a still better value of y_{i+1}. Thus process is repeated until we get 2 consecutive approximations of y_{i+1} are almost same. This method of refining an initially rough estimate by means of a more accurate formula is called a predictor method. The formula (1) is called predictor formula and formula (2)is called the correct formula.

5.1 Milne's Method

Consider $\frac{dy}{dx} = f(x, y)$with $y(x_0) = y_0$.

The value $y_0 = y(x_0)$ being given we compute $y_1 = y(x_0 + h), y_2 = y(x_0 + 2h), y_3 = y(x_0 + 3h) by Picard's or Taylor's series or Euler's method.$

Next we calculate ,

$f_0 = f(x_0, y_0), f_1 = f(x_0 + h, y_1), f_1 = f(x_0 + 2h, y_2), f_3 = f(x_0 + 3h, y_3)$

Then we find $y_4 = y(x_0 + 4h)$,we substitute in Newton's Forward Interpolation Formula.

Newton's Forward Interpolation Formula is

$$y = y_0 + \frac{u}{1!} \triangle y_0 + \frac{u(u-1)}{2!} \triangle^2 y_0 + \frac{u(u-1)(u-2)}{3!} \triangle^3 y_0 +$$

$$... + \frac{u(u-1)...(u-(n-1))}{n!} \triangle^n y_0$$

Now replace y by y'=$\frac{dy}{dx}$,then

$$y' = y'_0 + u \triangle y'_0 + \frac{u(u-1)}{2!} \triangle^2 y'_0 + \frac{u(u-1)(u-2)}{3!} \triangle^3 y'_0 + ... \quad (1)$$

Integrating w.r.t x in the interval$[x_0, x_0 + 4h]$

$$\int_{x_0}^{x_0+4h} y' dx = \int_{x_0}^{x_0+4h} [y'_0 + u \triangle y'_0 + \frac{u(u-1)}{2!} \triangle^2 y'_0 + ...] dx$$

$$[y(x)]_{x_0}^{x_0+4h} = h \int_0^4 [y'_0 + u \triangle y'_0 + \frac{(u^2-u)}{2!} \triangle^2 y'_0 + ...] du$$

[$\because$ when $x = x_0, u = 0, x = x_0 + 4h, u = 4$]

$$y(x_0 + 4h) - y(x_0) = h[y'_0 u + \frac{u^2}{2} \triangle y'_0 + \frac{1}{2}(\frac{u^3}{3} - \frac{u^2}{2}) \triangle^2 y'_0 + ...]$$

$$y_4 - y_0 = h[4y'_0 + 8 \triangle y'_0 + \frac{20}{3} \triangle^2 y'_0 + ...]_0^4$$

$$= h[4y'_0 + 8(y'_1 - y'_0) + \frac{20}{3}(y'_2 - 2y'_1 + y'_0) + \frac{8}{3}(y'_2 - 3y'_2 + 3y'_1 - y'_0)]$$

Neglecting higher order differences

$= 4\frac{h}{3}[2y'_1 - y'_2 + 2y'_3]$

$$y_4 = y_0 + 4\frac{h}{3}[2y'_1 - y'_2 + 2y'_3] \qquad (2)$$

Since x_0, x_1, x_2, x_3, x_4 are any 5 consecutive values of x,the above equation can be written generally as

$$y_{n+1} = y_{n-3} + 4\frac{h}{3}[2y'_{n-2} - y'_{n-1} + 2y'_n], n = 3, 4, ...$$

This is called Milne's Predictor's Formula.

T o obtain a corrector formula ,integrating (1) w.r.t x in the interval $[x_0, x_0+2h]$ we get

$$\int_{x_0}^{x_0+2h} y' dx = \int_{x_0}^{x_0+2h} [y'_0 + u \triangle y'_0 + \frac{u(u-1)}{2!} \triangle^2 y'_0 + ...]dx$$

$$[y(x)]_{x_0}^{x_0+2h} = h \int_{x_0}^{x_0+2h} [y'_0 + u \triangle y'_0$$

$$+\frac{u(u-1)}{2!} \triangle^2 y'_0 + \frac{u(u-1)(u-2)}{3!} \triangle^3 y'_0 + ...]du$$

$$y(x_0 + 2h) - y(x_0) = h[y'_0 u + \frac{u^2}{2} \triangle y'_0 +$$

$$\frac{1}{2}(\frac{u^3}{3} - \frac{u^2}{2}) \triangle^2 y_0' + ...]_0^2$$

$$y_2 - y_0 = h[2y_0' + 2 \triangle y_0' + \frac{1}{3} \triangle^2 y_0' + ...]$$

Neglecting higher orders we get

$$y_2 - y_0 = h[2y_0' + 2(y_1' - y_0') + \frac{1}{3}(y_2' - 2y_1' + y_0')]$$

$$\therefore y_2 = y_0 + \frac{h}{3}[y_0' + 4y_1' + y_2']$$

Since x_0, x_1, x_2 are consecutive values of x the above relation can be written generally as

$$y_{n+1} = y_{n-1} + \frac{h}{3}[y_{n-1}' + 4y_2' + y_{n+1}']$$

This is called Milne's Corrector Formula.
To apply Milne's Method ,we need four starting values of y.Hence this method is a multi-step method.

5.2 Milne Method-MATLAB

```
function[]=milne()
clc
clear all
format compact
format short g
global x y;
global h;
x=[0 0 0 0 0];
y=[0 0 0 0 0];
x(1)=input('enter the value of x0:');
xr=input('enter the last value of x:');
h=input('enter the spacing value:');
aerr=input('enter the allowed error:');
y=input('enter the value of y(i),i=0,3:');
for i = 1:3
x(i+1)=x(1)+i*h;
x(2:3)= x(2:3)+x(1,1)*6;
end
disp('x predicted corrected');
disp('x y f y f');
```

```
while(1)
if (x(4)==xr)
return
end
x(5)=x(4)+h;
y(5)=y(1)+(4*h/3)*(2*(f(2)+f(4))-f(3));
fprintf('%f %f %f \n',x(5),y(5),f(5));
correct();
while(1)
yc=y(5);
correct();
if(abs(yc-y(5)¡=aerr))
break;
end
end
for  i=1:4
x(i)=x(i+1);
y(i)=y(i+1);
end
end
function[z]=f(i)
z=x(i)+y(i);
end
function[]=correct()
```

```
y(5)=y(3)+(h/3)*(f(3)+4*f(4)+f(5));
fprintf('%f %f \n',y(5),f(5))
end
end
```

5.2.1 Example

Consider $\frac{dy}{dx} = x + y$ with initial condition y=1 at x=0.

By Picard's Method,$y = y_0 + \int_{x_0}^{x} f(x, y)dx$

$y = 1 + \int_{0}^{x}(x + y)dx$

The first approximation is

$$y_1 = 1 + \int_0^x (1 + x)dx$$

The second approximation is

$$y_2 = 1 + \int_0^x (1 + x + \frac{x^2}{2} + x)dx$$

$$= 1 + x + x^2 + \frac{x^3}{6}$$

The third approximation is

$$y_3 = 1 + \int_0^x (1 + x + \frac{x^2}{2} + x)dx$$

$$= 1 + x + x^2 + \frac{x^3}{3} + \frac{x^4}{24}$$

when $x = 0.2, y = 1.2200$

when $x = 0.4, y = 1.5707$

when $x = 0.6, y = 2.0374$

Now using Milne method (Matlab)

with h=0.2 we get

at x=0.8 ,y=2.6343

and at x=1,y=3.4287

x	y(Milne method)	y(actual method)
0.8	2.6343	2.6510
1.0	3.4287	3.4365

Chapter 6

AIRY'S EQUATION

6.1 Introduction and Importance

The standard form of an second order linear differential equation is $y''(t) + p(t)y'(t) + q(t)y(t) = g(t)$ where y(t) is the unknown function satisfying the equation and p , q and g are given functions, all continuous on some specified interval. We say that the equation is homogeneous if g =0.Thus:$y''(t) + p(t)y'(t) + q(t)y(t) = 0$. The equation is said to be a constant coefficient equation if the functions p and q are constant. Non-constant coefficient equations are more problematic. The series solutions method is used primarily, when the coefficients p(t) or q(t) are non-constant.

Airy equation or the Stokes equation is the simplest second-order linear differential equation with a turning point. We define a general Airy's equation as $y'' \pm k^2xy = 0$. In the physical sciences, the Airy function (or Airy function of the first kind) Ai(x) is a special function named after the British astronomer

George Biddell Airy (1801-92). The function Ai(x) and the related function Bi(x), called the Airy function of the second kind are sometimes referred to as the Bairy function, are linearly independent solutions to the differential equation. The Airy function of the first kind can be defined by the improper Riemann integral:Ai(x)=$\frac{1}{\Pi}\int_0^\infty cos(\frac{t^3}{3}+xt)dt$ and the Airy function of the second kind Bi(x)=$\frac{1}{\Pi}\int_0^\infty[exp(\frac{-t^3}{3}+xt)+sin(\frac{t^3}{3}+xt)]dt$
The frequent appearances of the Airy functions in both classical and quantum physics is associated with wave equations with turning points. Within classical physics, they appear prominently in physical optics, electromagnetism, radiative transfer, fluid mechanics, and nonlinear wave propagation. Extensive use is made of Airy functions in investigations in the theory of electromagnetic diffraction and radiowave propagation. The Airy function is the solution to Schrödinger's equation for a particle confined within a triangular potential well and for a particle in a one-dimensional constant force field. The Airy function is also important in microscopy and astronomy; it describes the pattern, due to diffraction and interference, produced by a point source of light.

6.2 Solution of Airy's Equation using Power Series

There are different ways for solving airy's equation like power series method,WKB method(asymptotic expansion),etc.Here we solve by power series method and get its graph using Matlab. The generic form of a power series is $y(x) = \sum_{n=0}^{\infty} a_n x^n$.

$$Then y'(x) = \sum_{n=1}^{\infty} n a_n x^{n-1} = \sum_{n=0}^{\infty} (n+1) a_{n+1} x^n$$

.

$$y''(x) = \sum_{n=0}^{\infty} n(n+1) a_{n+1} x^{n-1} = \sum_{n=0}^{\infty} (n+1)(n+2) a_{n+2} x^n.$$

Now setting $k^2 = 1$ we get $y'' - xy = 0$. Substituting y'' in the differential equation we get

$$\sum_{n=0}^{\infty} (n+1)(n+2) a_{n+2} x^n - x \sum_{n=0}^{\infty} a_n x^n = 0$$

$$\sum_{n=0}^{\infty} (n+1)(n+2) a_{n+2} x^n - \sum_{n=0}^{\infty} a_n x^{n+1} = 0$$

.

$$2a_2 + \sum_{n=1}^{\infty} (n+1)(n+2) a_{n+2} x^n - \sum_{n=1}^{\infty} a_{n-1} x^n = 0$$

$$2a_2 + [\sum_{n=1}^{\infty} (n+1)(n+2) a_{n+2} - a_{n-1}] x^n = 0$$

The power series on the left is identically equal to zero, consequently all of its coefficients are equal to 0:

$2a_2 = 0$

$(n+1)(n+2)a_{n+2} - a_{n-1} = 0$ for all n=1,2,3,...

$\therefore a_2 = 0, a_{n+2} = \frac{a_{n-1}}{(n+1)(n+2)}$ for all n=1,2,3,... These equations are known as recurrence relations of the differential equations.The recurrence relations permit us to compute all coefficients in terms of a_0 and a_1. When n=1, $a_3 = \frac{a_0}{2.3}$

Continue giving values to n to get

$a_4 = \frac{a_1}{3.4}$

$a_5 = \frac{a_2}{4.5} = 0$

$a_6 = \frac{a_3}{5.6} = \frac{a_0}{(2.3)(5.6)}$

$a_7 = \frac{a_4}{6.7} = \frac{a_1}{(3.4)(6.7)}$

$a_8 = \frac{a_5}{7.8} = 0$

$a_9 = \frac{a_6}{8.9} = \frac{a_0}{(2.3)(5.6)(8.9)}$

So when can see that

1)All the terms $a_2, a_5, a_8, ...$ are equal to zero.

$\therefore$ a_{3k+2}=0 for all k=0,1,2,3,...

2)All the terms $a_3, a_6, a_9, ...$ are multiples of a_0 .

$\therefore$ $a_{3k} = \frac{a_0}{(2.3)(5.6)...((3k-1).3k)}$ for all k=1,2,3,...

3)All the terms $a_4, a_7, a_{10}, ...$ are multiples of a_1.

$\therefore$ $a_{3k+1} = \frac{a_1}{(3.4)(6.7)...(3k.(3k+1))}$ for all k=1,2,3,...

Thus the general form of the solution to Airy's Equation is given by

$$y(x) = a_0[1+\sum_{k=1}^{\infty} \frac{x^3k}{(2.3)(5.6)...((3k-1).3k)}]+a_1[x+\sum_{k=1}^{\infty} \frac{x^{3k+1}}{(3.4)(6.7)...(3k.(3k+1))}]$$

As always we have $y(0) = a_0$ and $y'(0) = a_1$.Thus it is trivial to determine a_0 and a_1 when we want to solve an initial value condition.

In particular

$$y_1(x) = 1 + \sum_{k=1}^{\infty} \frac{x^3k}{(2.3)(5.6)...((3k-1).3k)}]$$

and

$$y_2(x) = [x + \sum_{k=1}^{\infty} \frac{x^{3k+1}}{(3.4)(6.7)...(3k.(3k+1))}]$$

forms a fundamental system of solutions for Airy's equation.

6.3 Airy's equation and Matlab

Now as we seen that airy's equation is important in physical sciences, Matlab has inbuilt airy's equation solution. Also by solving airy's equation in Matlab using dsolve command :

syms y(x)

dsolve('D2y-x*y=0','x')

we get the output as C2*airy(0, x) + C3*airy(2, x)

where airy(0,x) is the airy's function of first kind and airy(2,x) is the airy's fuction of second kind.

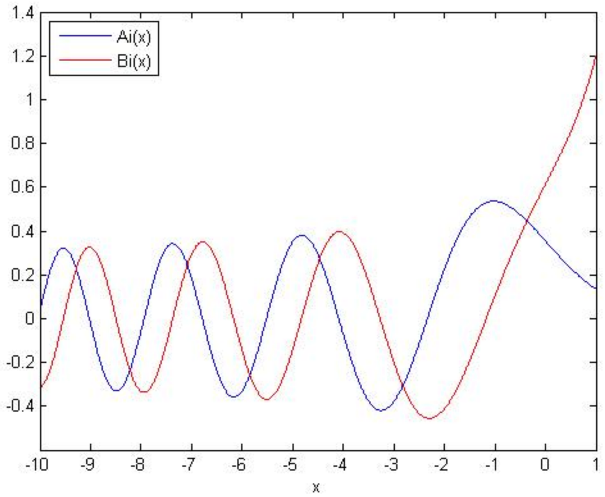
Also we can get the values these 2 airy's function .For example if we want the value of airy's function of 1st kind and 2nd kind at the point say 1.5 say, then can we use the Matlab command airy(0,1.5) and airy(2,1.5). Also we can the values of the derivatives of airy's function of two kinds in Matlab.The derivative of airy's function of first kind is represented by airy(1,x) and derivative of airy's function of second kind is represented by airy(3,x).

Now we can plot the solution of airy's equation easily in Matlab.

The Matlab command for obtaining the graph of this is as follows:

```
x = -10:0.01:1;
%Calculate Ai(x)
ai = airy(x);
%Calculate Bi(x) using k = 2.
bi = airy(2,x);
%Plot both results together on the same axes.
figure
plot(x,ai,'-b',x,bi,'-r')
axis([-10 1 -0.6 1.4])
xlabel('x')
legend('Ai(x)','Bi(x)','Location','NorthWest'
```

The graph obtained is as follows:



Chapter 7

PERTURBED EQUATIONS

7.1 Introduction

The study of perturbation equations is important as they arise in various branches of engineering and applied mathematics. A perturbation is a 'disturbance', usually a small disturbance. Perturbation method is a method for obtaining approximate solution to algebraic and differential equations for which exact solution is not easy to find. Consider an equation, or a system, and say we know the solution of the system. If the equation changes very slightly (perturbed), perhaps the solution will also change slightly. In that case, we can find the solution of the 'perturbed equation' to be very close to the solution of the unperturbed equation.Mainly such problems which contain at least one small parameter ε known as perturbation parameter. We usually denote ε for the effect of small disturbance in physical system and $0\leq \varepsilon < 1$.

7.2 Regularly perturbed equation

Depending upon the nature of perturbation, a perturbed problem can be divided into two types: Regularly perturbed and Singularly perturbed.

A perturbation equation is said to be regular if the degree of the perturbed and unperturbed remain the same when $\varepsilon = 0$.

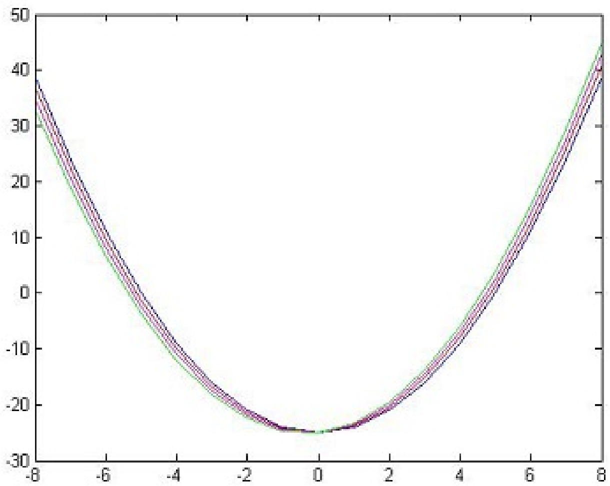
7.2.1 Example 1

Consider the quadratic equation $x^2 + \varepsilon x - 25 = 0$

When $\varepsilon = 0$,the equation becomes $x^2 - 25 = 0$

Now we use Matlab for plotting the graph of this for different values of ε

The graph below shows the graph of the quadratic equation for ε=0(blue), ε=.25(red),ε=.5(magenta)and ε=.75(green).



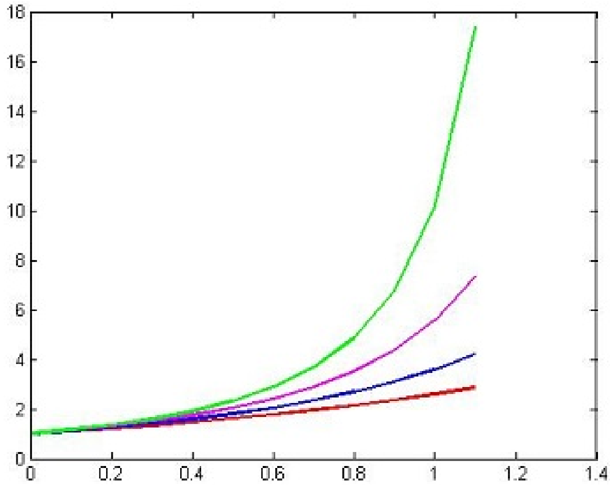
7.2.2 Example 2

Consider the differential equation $\frac{dy}{dx} = y + \varepsilon y^2$ with y(0)=1.

When ε=0 ,the equation becomes $\frac{dy}{dx} = y$.

Now we solve the perturbed equation for different values of ε using euler's method. And we can plot them using Matlab.

The graph for ε =0(red), ε=0.2(blue) , ε=0.4(magenta) and ε=0.6(green) is as shown below.



7.3 Singularly perturbed equation

A perturbed equation is said singularly perturbed if the degree of equation is reduced when ε =0. Generally, the parameter presented at higher order terms and the lower order terms starts to dominate.

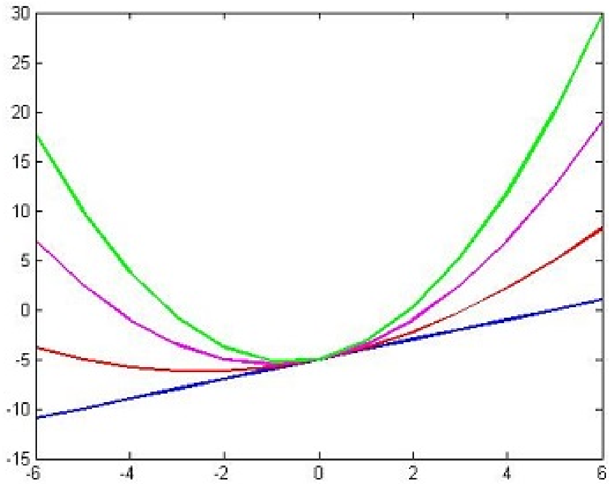
7.3.1 Example 1

Consider the quadratic equation $\varepsilon x^2 + x - 5 = 0$

When ε=0 the equation becomes x-5=0.

Now we use Matlab for plotting the graph of this for different values of ε.

The graph below shows the graph of the quadratic equation for ε=0(blue one), ε=.2(red one),ε=.5(magenta)and ε=.8(green).



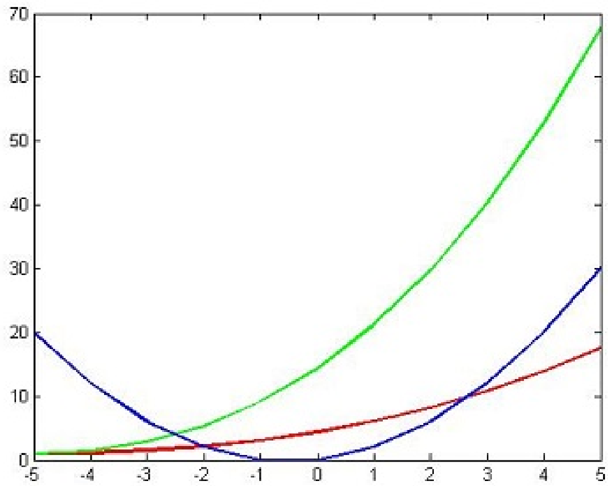
7.3.2 Example 2

Consider the differential equation $\varepsilon \frac{dy}{dx} = y + y^2$

When ε=0 , the nature of the equation itself changes. It becomes a quadratic equation.

Now we solve the perturbed equation for different values of ε using R K method and plot them using Matlab.

The graph for ε=0(blue), ε=.2(red),ε=.8(green) is as shown below.



Thus with the help of numerical methods and Matlab we can study perturbation equations and see the variations that happen due to the change the value of the parameter ε.

CONCLUSION AND FUTURE WORK

In this project, we have discussed different numerical methods for finding the solution of ordinary differential equations with initial conditions. We can see that Modified Euler's method and R K Method gives the best solution and R K Method is the most widely used method among all these methods. So, we can conclude that these numerical methods are useful for finding an approximate solution of differential equations. We have also discussed an important equation in physical science , Airy's equation and plot its graph with the help of Matlab. In the last chapter, we have discussed the some basics concepts of perturbation equations with the help of numerical method and Matlab.So we can conclude that Matlab is very helpful for plotting graphs and reduces our work and time.

This work can be extended in various directions:

The problems we solve here are first order differential equations. We can extend this work by discussing numerical methods for higher order differential equations and program them in Matlab which will be very useful in reducing our time.

We have only introduced the basic concepts of perturbed equations and we can extend this by solving more complex perturbed equations by constructing new numerical methods or modifying existing numerical methods and designing suitable Matlab programs for these methods.

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