A STUDY ON DOMINATION IN GRAPH THEORY

A Dissertation submitted in partial fulfillment of the

requirementfor the award of

DEGREE OF MASTER OF SCIENCE

IN MATHEMATICS

By

ANJALY.T.S

REGISTER NO: SM16MAT004

(2016 – 2018)

DEPARTMENT OF MATHEMATICS

ST.TERESA'S COLLEGE, (AUTONOMOUS)

ERNAKULAM, KOCHI - 682011

APRIL 2018

DEPARTMENT OF MATHEMATICS

ST.TERESA’S COLLEGE (AUTONOMOUS), ERNAKULAM

CERTIFICATE

This is to certify that the dissertation entitled “A STUDY ON DOMINATION IN GRAPH THEORY”is a bonafide record of the work done by ANJALY.T.S under my guidance as partial fulfillment of the award of the degree of Master of Science in Mathematics at St.Teresa’s College (Autonomous), Ernakulam affiliated to Mahatma Gandhi University, Kottayam. No part of this work has been submitted for any other degree elsewhere.

Smt.Dhanalakshmi.O.M (Supervisor)

Assistant Professor

Department Of Mathematics

St. Teresa’s College, (Autonomous)

Ernakulam

Smt Teresa Felitia (HOD)

Associate Professor

Department of Mathematics

St. Teresa’s College (Autonomous)

Ernakulam

External Examiners:

1..

2..

DECLARATION

I hereby declare that the work presented in this project is based on the original work done by me under the guidance of Smt DHANALAKSHMI.O.M, Assistant Professor, Department of Mathematics; St Teresa's College (Autonomous) Ernakulam and has not been included in any other project submitted previously for the award of any degree.

ERNAKULAM | ANJALY.T.S
APRIL 2018 | **REGISTER NO: SM16MAT004**

ACKNOWLEDGEMENT

I have great pleasure in extending my own wholehearted thanks to Smt.Dhanalkshmi.O.M ,my guide assistant professor ,for presenting the project report.

I wish to express my sincere thanks to Smt. Teresa Felitia , Head of the department, St.Teresas College,Ernakulam for the complete facilities made available for this project.

My thanks to all members of the department of mathematics for their wholehearted support and help.

I wish to express thanks to all my companions for their help and encouragement to bring the project a successful one.

ERNAKULAM ANJALY.T.S

APRIL 2018 **REGISTER NO: SM16MAT004**

CONTENTS

INTRODUCTION

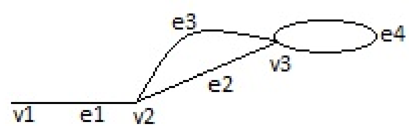
Domination in graphs has been extensively researched branch of graph theory.Graph theory is one of the most flourishing branches of modern mathematics.The last 30 years have with one and spectacular growth of graph theory due to its wide application to discrete optimization problems,combinotorial problems and classical algebraic problem.It has wide range of physical,social and biological sciences;linguistic etc,the theory of domination has been the nucleus of research activity in graph theory in recent times.This is largely due to the variety of new parameters that can be developed from the basic definition of domination.

The rigorous study of dominating set in graph theory began around 1960,even though the subject has historical roots dating back to 1862 when de jaenisch studies the problem of determining the minimum number of queens which is necessary to cover or dominate an nxn chessboard.In 1958,berge defined the concept of domination number of a graph,calling this as ”coefficient of external stability”.In 1962,ore used the name ”dominating set” and domination number for the same concept.In 1977 cockayne and extensive survey of results know at that time about dominating set in graph.They have used the notation (G) for the domination number of graph,which has become very popular since then.The survey paper of cockayne and hedetnieme has generated a lot of interest in study of domination in graphs .In a span about twenty years after the survey more than 1200 research papers have been published on this topic.

In this chapters describes about domination insets,more about varieties of domination,common minimal domination etc and theorms,results related to it and application of domination in graphs.

PRELIMINARY CONCEPTS

Graph:
A graph is an ordered triple=(V(G),E(G),I_G) where V(G) is a non empty set,E(G) is set disjoint from V(G) and I_G is incident map that associates with each elements of E(G) and ordered pair of elements(same or distinct of V(G)).
Elements of V(G) are called vertices/nodes/points and elements E(G) are called edges or lines of G.
If E is an edge and u and v are vertices such that I_G(e)=uv;then e is said to join u and v where the vertices u and v are called ends of e,then we say e is incident with the end also the vertices u and v are incident with e.



V(G)=$\{v1, v2, v3\}$
E(G)=$\{e1, e2, e3, e4\}$
I_G(e1)=v1 v2
I_G(e2)=v2 v3
I_G(e3)=v3 v2
I_G(e4)=v3 v3

Subgraph:
A graph H is a subgraph og G (H⊆G) if V(H)⊆V(G),E(H)⊆E(G) and I_H is a restriction of I_G to E(H).
A spanning subgraph of G s a subsraph with V(H)=V(G).

Parallel edges:
Two or more edges of a graph G with same end is called parallel edges.
Eg:e2,e3.

Loops:
An edge with identical end is called loops
Eg:e4

Link:

An edge with distinct end is a link.
Eg:e1,e2,e(not loops)

Neighbourhood:
A vertex u is a neighbor of v in G if uv is an edge of G and u≠v.
The set of all neighbors of v is the open neighborhood of v denoted by N[V].
The set N[V] is closed neighbourhood of v in G ,N[V]=N(v)$\cup\{v\}$.

Adjacency:
Two vertices u and v in G are said to be adjacent iff if there is an edge of G with u and v as its ends.
Two distinct edges e and f are said to be adjacent iff they have a common end vertex.

Simple graph:
A graph is simple if it has no loops and no parallel edges.

Finite and infinite graph:
A graph is finite if both E(G) and V(G) are finite.Otherwise it is infinite.

Order:
The number of vertices of graph G is called the order of the graph denoted by n(G).

Size:
Number of edges of a graph G is called size of graph denoted by m(G) or simply n.

Degree of vertices:
Let G be a graph and vϵV.The number of edges incident at v in G ia called degree of vertexv in G and dented by d_G(V).
The minimum(respectively maximum) of degrees of a graph G is denoted by δ(G) or simply δ (respectively $\triangle$(G) or $\triangle$).

Regular graph:
A graph is called K regular graph if every vertex G has degree K.A graph is said to be regular if it is K-regular for some non zero K.

Isolated vertex:
It is a vertex with degree zero.That is,it is a vertex which not an end point of any edge.

Leaf:
A leaf vertex (also pendent vertex) is a vertex with degree one.

Walk:
A walk in graph G iv_0e_1s alternating sequence W:v_0e_1,v_1e_2.....$v_{n-1}e_nv_n$ of vertices and edges , begnning and ending with vertices in which v_{i-1} and v_i is the origin and v_n is terminus of W.The walk W is said to join v_0 and v_n is called v_0-v_n walk.

Path:
A path in graph is an open walk in which no vertex and therefore no edges is repeated.

Trail:
A trail in a graph is a walk in which no edge is repeated.

Cycle:
A closed walk in which no vertex (and edge) is repeated is called a cycle.

Connected:
Two vertices u and v are said to be connected if there is a uv path in G.

Euler Trail:
A trail that traverses every edge of G is called an Euler trail.

Euler Tour:
A tour of G is a closed walk that traverses each edge of G atleast once.
A euler tour of G is the tour which traverses each edge exactly once or it is a closed euler trail.

Euler graph:
A graph is eulerian if it contains a euler tour.

Hamiltonian path:
A path that contain every vertex of G or it is a path in G in which every vertex is included exactly once.

Hamiltonian cycles:
A cycle that contain every vertex of G is called a hamiltonian cycle of G or any closed hamiltonian path in G is a hamiltonian cycle in G.

Hamiltonian graph:
A graph is hamiltonian if it contain a hamiltonian cycle.

Spanning tree:
A spanning tree T of an undirected graph G is a subgraph that is a tree which includes all vertices of G,with minimum possible number of edges.

Tree:
Any acyclic connected graph is a tree.

Matching:
A matching M in G is set of pairwise non adjacent to each other.
A matching graph G is a subgraph of a graph G where there is no edges adjacent to each other.

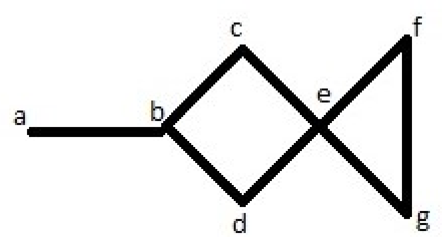
Perfect matching:
A perfect matching is a matching containing edges (the largest possible meaning of perfect matching are only possible on graphs with an even number of vertices).

CHAPTER-1
DOMINATING SET

Definition-1
Let G=(V,E) be a graph.A set D⊆V is called dominating set of G if every vertex u∈V-D has a neighbor v∈D

Defnition-2
A set D⊆V of vertices in G=(V,E) is dominating set if N_G[D]=V(G) ie,If every vertex in V(G) is either in D or adjacent to a vertex in D

Defnition-3
A dominating set for graph G-(V,E) is a subset D of V such that every vertex not in D is adjacent to atleast one member of D



eg:
For instance the vertex set {b,g} is dominating set in this graph.The set {a,b,c,d,f} is dominating set of the graph G.

Minimal Dominating set
A dominating set D of the graph G is said to be a minimal dominating set if for every vertex v in D,D-{v} is not a dominating set.That is no proper subset of D is a dominating set
In figure 1,{b,e} and {a,c,d,f} are minimal dominating set.

Minimum dominating set
A dominating set with least number of vertices is called Minimum dominating set
eg:{b,g} in figure 1

Domination number
The number of vertices in a minimum dominating set is called domination

number of graph G.It is denoted by γ(G)
eg:The Minimum dominating set {b,e} in figure 1 has 2 elements.so γ(G)=2.

Theorm 1.1

Let S be a dominating set of a graph G.Then S is minimal dominating set of G iff for every vertex u∈s one of the following conditions is satisfied
(i) u is a isolate of S ie,(N(u))∩S=Φ
(ii)there exist a vertex v∈V-S(N(u))∩S={u})

proof:
First we suppose that S is minimal dominating se of G.Let u∈S.Then S-{u} is not dominating set of G.Hence there exist v∈V-{$S-${u}}=$(V-U)$∪{u} such that,(N(v))∩{$S-${u}}=Φ
If v=u, u satisfies condition (i)
If v≠u, we then have (N(v))∩{$S-${u}}=Φ and(N(v))∩S≠Φ.since S is a dominating set.Then (N(u))∩S={u} and condition (ii) satisfied.
Conversely,Let us suppose that S is not a minimal dominating set of G.ie,there exist S'⊆S such that S' is dominating set of G.Then there exist u∈S-S'.Then S'⊆S-{u}⊆S.Since S' is a dominating set,so is S-{u}.Then as u∈S-{u},it must be adjacent to a vertex in S-{u}.ie,u is not an isolate of S.Let v∉S,then (N(v))∩{$S-${u}}=Φ
ie, (N(u))∩S≠{u}.Hence either condition (i) and (ii) is satisfied for this particular u∈S.

Theorm 1.2:

Let G be a graph with no isolated vertices and S be a minimal dominating set of G.Then V-S is also a dominating set of G.

Proof:
Let us consider that u∈S.The claim is N(u)⊈S.Let us suppose that the contrary where N(u)⊆S.Then u must be adjacent to a vertex in S and any vertex in V-S must be adjacent only to vertex in S-{u}.This implies S-{u} is a dominating set,a contradiction to minimality of S.
Then for any u∈S ,N(u)⊈S
equalently N(u) ∩ V-S≠ϕ.
⇒V-S is dominating set.

Corollary:

If G is a graph of order n without isolated vertices then γ(G)≤$\frac{n}{2}$.

Proof:
Let S be minimum dominating set of G.Thus |S|=γ(G).By above theorm,V(G)-S ia a dominating set of G and so
γ(G)=|S|$\leq$|V(G)-S|=n-γ(G). therefore, 2γ(G)$\leq$n $\Rightarrow$ γ(G)$\leq\frac{n}{2}$.

CHAPTER 2

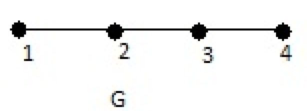
VARIETIES OF DOMINATION

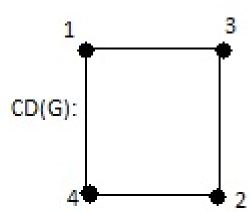
Section 2.1

COMMON MINIMAL DOMINATION

Definition:
Let G=(V,E) be a graph.A set D⊆V is the dominating set.A dominating set D of G is **minimal** if for any vertex v ϵ D,D-v is not dominating set of G.The minimum and maximum cardinalities taken over minimal dominating set of G are domination number γ(G) and upper domination number Γ(G) of respectively.
The neighbourhood graph N(G) of a graph G is having the same vertex set as G with two vertices adjacent in N(G) iff they have a common neighbor in G.
Now we define a similar type of graph namely "The common minimal dominating graph "as follows:
The **common minimal dominating graph CD(G)** of graph G is graph having the same vertex set as G with two vertices adjacent in CD(G) iff if there exist a minimal dominating set in G containing them.

In below figure , a graph G and its common minimal dominating graph CD(G) are shows:





Let G' be complement of G.

Theorm 2.1.1:
For any graph G,
G'⊆CD(G) and G'=CD(G) iff every minimal dominating set of G is independent.

Proof:
If (uv)ϵ E(G'), then extended $\{u, v\}$ to maximal independent set S of vertices in G.Since S is also a minimal dominating set of G,we obtain G'$\subseteq$CD(G).
Now we prove second part.
If every minimal dominating set of G is independent ,then two vertices adjacent in G cannot be adjacent in CD(G).Thus CD(G)$\subseteq$G' and together with G'$\subseteq$CD(G), we see that CD(G)=G'.
Coversely,
CD(G)$\subseteq$G'
$\Rightarrow$Two vertices in same minimaldominating set S are not adjacent in G.
That is , S is independent.

Let $\triangle$(G) denote the maximum degree of G.

Theorm 2.1.2:
For any graph with p vertices p$\geq$2,CD(G) is connected iff $\triangle$(G) < p-1

Proof:
Let $\triangle$(G) < p-1 and u,v be any two vertices of G.
If (u,v)$\notin$E(G),then by theorm 2.1,(u,v)$\in$CD(G).
If (u,v)$\in$ E(G) and some vertex w distinct from both u and v is adjacent to neither ,then again by theorm 2.1.1,in CD(G) the path uwv joins u to v.It only remains to consider (u,v)$\in$E(G) and every other vertex w is adjacent to atleast one of u and v.Then $\{u, v\}$ is minimal dominating set of G and hence (u,v)$\in$CD(G).Thus CD(G) is connected.
Conversely,
Suppose CD(G) is connected.If possible,suppose $\triangle$(G)=p-1 and u is a vertex of degree p-1.Then u is isolated vertex in CD(G).Since G has atleast two vertices CD(G) has atleast two components , a contradiction.Thus $\triangle$(G)<p-1.

Note:
In a graph G, avertex and an edge incident with it are said to cover each other.A set of vertices which covers all the edges is a vertex cover of G.The vertex covering number α_0(G) in G is minimum number of vertices in a vertex cover.A set S of vertices in G is independent if no two vertices in S are adjacent.The independent number β_0(G) of G is the maximum cardinality of an independent set of vertices.

Theorm 2.1.3:
For any graph G,γ(CD(G))$\leq\omega$(G).

Proof:
By theorm 2.1.1,
γ(CD(G))$\leq\gamma$(G)
$\leq\beta_0$(G)
$\leq\omega$(G).

Theorm 2.1.4:
For any graph G,γ(CD(G))$\leq$p-Γ(G)+1.

Proof:
Let S be a minimal dominating set with |S|=Γ(G).Then γ(CD(G))$\geq\Gamma$(G).Thus γ(CD(G))$\leq$p-Γ(G)+1 follows from the fact that γ(G)$\leq$p-Δ(G) and $\triangle$(G)$\geq\omega$(G)-1.For ant graph G,let α0(G) denote the vertex covering number of G. That is the minimum number of vertices in a vertex cover and use the fact that α_0(G) + β_0(G)=p.

Theorm 2.1.5:
For any graph G,γCD(G)$\leq$1+δ(G).

Proof: By theorm 2.1.1,$\triangle$CD(G)$\geq\triangle$(G')=p-1-δ(G).
Hence γCD(G)$\leq$1+δ(G) follows from the first fact used in proof of theorm 2.1.4.

To prove th next two results we make use of the following results from Harary.
Theorm A: A graph G is eulerian iff every vertex of G is of even degree.
Theorm B: If every vertices v of G, deg(v)$\geq\frac{p}{2}$ where p $\geq$,then G is hamiltonian.
A graph G is odd iff every vertex of G is of odd degree.
Let diam(G) denote the diameter of G.

Theorm 2.1.6:
If G is an odd graph with Γ(G)=diam(G)=2,then CD(G) is eulerian.

Proof:
Since p is even,every vertex in g is non adjacent to an even number of vertices.Hence every vertex in G' of even degree and by theorm A,G' is eulerian.Since for any two vertices u,v in G.there exist a vertex w which is not adjacent to both u and v and further there is no minimal dominating set in G containing u and v ,by theorm 2.1.1, CD(G)=G'
Hence CD(G) is eulerian.

Let [x] denote the greatest integer not greater than x.

Theorm 2.1.7:
Let G be a graph of order atleast three satisfying one of the following conditions.
(i)$\triangle$(G)<$[\frac{p}{2}]$;
(ii)$\triangle$(G)=$[\frac{p}{2}]$ and for every vertex v of G with deg(v)=$[\frac{p}{2}]$,there exist a vertex u $\in$ N(v) such that u is adjacent to every vertex in V-N(v).
Then CD(G) is hamiltonian.

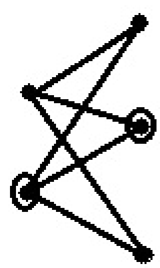
Proof:
Suppose (i) holds.Then δ(G')$\geq\frac{p}{2}$ and hence by G'$\subseteq$CD(G) and G'=CD(G) in theorm 2.1.1, and theorm B ,CD(G) is hamiltonian.
Suppose (ii) holds.Then $\{u, v\}$ is dominating set of G and further it is minimal,since there exist two vertices u_1 $\in$N(u)-N(v) and v_1 $\in$N(v)-N(u).Hence by G'$\subseteq$CD(G) and G'=CD(G) in theorm 2.1.1,deg(v) in CD(G) $\geq\frac{p}{2}$.Also by (i), for any vertex u with deg(u)<$[\frac{p}{2}]$,deg(u) in CD(G) $\geq\frac{p}{2}$.Hence by theorm B,CD(G) is hamiltonian.

Section 2.2

TOTAL DOMINATION

Definition:
A set S⊆V is a **total dominating set** if every vertex in V is adjacent to some vertex of S.
Alternatively , we may define a dominating set D to be a total dominating set if G[D] has no isolated vertices.
The **total domination number** of G,denoted by γ_t=γ_t(G) is cardinality of smallest dominating set,and we refer to such a set as a γ_t-set.An immediate consequence of definitions of domination number and total domination number is that , for any graph G,$\gamma_t \geq \gamma$.

An example of equality in domination and total domination: $K_{2,3}$



Some basic result for domination and total domination

Theorm 2.2.1:
Let G be a graph with no isolated vertices.Then $\gamma \leq \frac{n}{2}$.
Proof:
Let D⊆V(G) be a γ-set.Since G has no isolated vertices ,every vϵD has atleast one neighbor in V-D.This means that V-D is also a dominating set.If |D| $> \frac{n}{2}$,then V-D is a smaller dominating set,contradicting the choice of D as a γ set.Thus γ= |D| $\leq \frac{n}{2}$.

Theorm 2.2.2:
Every graph G with no isolated vertices has a γ-set D such that svery v ϵ D has the property that there exist a vertex v'ϵV-D that is adjacent to v but to no other vertices of D.

The vertex v' from Theorm 2.2.2 called a private neighbor of v.from now on we will assume that every γ-set is chosen to be one whose existance is guarenteed by theorm 2.This allows us to prove the following triple inequality.

Theorm 2.2.3:
Let G be a graph with no isolated vertices.Then $\gamma \leq \gamma_t \leq 2\gamma$.

Proof:
The first inequality follows immediately from definition of domination number and total domination number.Let D be a γ-set.If D is not a total dominating set , it is because the subgraph induced by D has isolated vertices.By theorm 2.2.2 , each of these isolated vertices has a private neighbor .To construct a dominating set , we simply add D to private neighbor for each isolated vertex and call the new set as D'.Atmost |D| private neighbor could have been added to form D'.That is $|D'| \leq 2|D|$.since is a total dominating set,$|D| \geq \gamma_t$.
Therefore,$\gamma_t \leq 2|D| = 2\gamma$.This proves the second inequality.

Theorm 2.2.4:
Let G be a graph of order n with no isolated vertices.Then $\gamma_t \geq \dfrac{n}{\triangle}$

Proof:
Let S be a γ_t - set. Then by definition ,every vertex of G is adjacent to some vertex of S.That is,N(S)=V(G).since every vertex v ϵ S can have atmost $\triangle$ neighbors,it follows that $\triangle\gamma_t \geq |V| = n$.The theorm follow by dividing this inequality by $\triangle$.

Section 2.3

INDEPENDENT DOMINATION

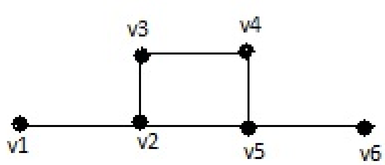
Definition:

A set S⊆V is **independent** if no two vertices of S are joined by an arc. An independent set S is said to be **maximum independent** set if any vertex set properly containing S is not independent.

A subset S of the vertex of a graph G is an **independent dominating set** if both an independent and dominating set.

Independent domination number:
The independent domination number i(G) of G is minimum cardinality of an maximum independent dominating set.
Fig A:

γ(G)=$\{2, 4\}$
i(G)=$\{2, 4\}$
therefore,γ(G)=2=i(G).
Fig B:



γ(G)=$\{v5, v2\}$
i(G)=$\{v3, v5, v1\}$
γ(G)=2 while i(G)=3

It is Clear that γ(G)≤i(G),for any graph G
For the path p_5,γ(p_5)=i(p_5)(Fig A).While for the graph of G of G of Fig B,γ(G)=2 and i(G)=3.Infact $\{v5, v2\}$ is a γ-set for G,While $\{v3, v5, v1\}$ is a minimum independent dominating set of G.

Theorm 2.3.1:
For any graph G,i(G)+i(G')$\leq$n-$\triangle$+δ+1.
proof:
we know for any graph G,i(G)$\leq$n-$\triangle$
therefore,i(G)+i(G')$\leq$n-$\triangle$+i(G')
=n-$\triangle$+n-(n-1-δ)
=n-$\triangle$+δ+1.
therefore,i(G)+i(G')$\leq$n-$\triangle$+δ+1.

Theorm 2.3.2:
If G is a K regular graph k¿0 then i(G)$\leq\frac{n}{2}$.
Proof:
with out loss of generality one can assume that G is connected.Let S be an i(G)-set.if i(G)=|s|>$\frac{n}{2}$,Then |V-S|<|s|.By assumption ,we have δ=$\triangle$ and so there exist atleast one vertex v in V-S such that d(v)>$\triangle$,which is a contradiction.
hence the theorm.

Theorm 2.3.3:
For isolated free graph G and G',i(G)+i(G')$\leq$n.
proof:
If G is regular then by theorm 2,i(G)$\leq\frac{n}{2}$ and i(G)+i(G')$\leq$n.on other hand ,if G is not regular,then $\triangle$-$\delta\geq$1 and hence by theorm 1,i(G)+i(G')$\leq$n.

Corollary:
i(G)+i(G')=n+1 implies either G or G' is complete.
Proof:
Let i(G)+i(G')=n+1 .
From theorm 2.3.3 G and G'must contain isolated vertices.
By theorm 1,n+1 =i(G)+i(G'))$\leq$n-$\triangle$+δ+1.
$\Rightarrow$ $\triangle$=δ
Thus either G or G' is complete.
Converse is trivial.

Theorm 2.3.4:
Every maximal independent set of graph G is minimal dominating set.
Proof:
Let S be a maximal dominating Set.Then G must be a dominating set.If not ,there exist a vϵV/S that is not dominating by S and so S$\cup\{v\}$ is an

independent set of G,violating the maximality of S.Further ,S must be a minimal dominating set of G.If not ,there exist a vertex u of S such that T=S/$\{u\}$ is also a dominating set of G.This means that as u not belonging to T,u has a neighbor in T and hence S is not independent,a contradiction.Hence Every maximal independent set of graph G is minimal dominating set.

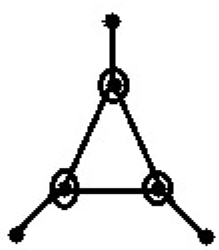
Section 2.4

CONNECTED DOMINATION(CDS)

Definition:
consider a graph G=(V,E).Let D be dominating set.If D induces a connected subgraph,then it is called a connected dominating set.

Connected domination number:
A connected domination number of a graph G is minimum cardinality of connected dominating set.It is denoted by γ(G).
A CDS that has size equal to domination number is called a minimum CDS.

An example of equality in domination,total domination,connected domination:



Let ℓ(G) denote the maximum leaf number of a graph G,which is maximum number of leaves in a spanning tree.

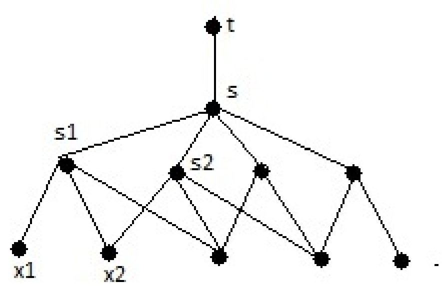
Theorm 2.4.1
For any graph of order n,γ_c=n-ℓ(G).

proof:
It is easy to see that for any tree T,γ_c(T)=| V(T)-ℓ(T)|.
More over , a CDS for spanning tree T of G is also a CDS for G.
therefore,

$$\gamma_c \leq \text{n-}\ell\text{(G)}.$$

Now consider a minimum CDS D of S.Let H be a spanning tree of G[D],where G[D] is subgraph of G induced by D.Connect H to every vertex in V-D to obtain a spanning tree of G.Then every vertex in V-D is a leaf of T.
Conversely,
Every leaf of T is in V-D.Otherwise if t has a leaf x not in T-D,than D-$\{x\}$ would be a CDS for T and hence a CDS for G,contradicting the minimality of D.

Graph of G in proof of theorm given below:



Theorm 2.4.2:
Let G be a graph Of Order n$\geq$4.Suppose both graph G and its complement G' are connected.Then γ_c(G)+ γ_c(G') $\leq$ n(n-3)

Proof:
We know that a tree has atleast 2 leaves.By above Theorm,
γ_c(G) $\leq$ n-2
Moreover, G is connected and hence
n-1 $\leq$ |E(G)|
therefore γ_c(G) $\leq$ n-2=2(n-1)-n $\leq$ 2|E(G)|-n
similarly,

γ_c(G') $\leq$ 2|E(G')|-n
Thus,
γ_c(G)+ γ_c(G') $\leq$ 2(|E(G)|+|E(G')|)-2n
=2*nC_2 =2n =n(n-3).

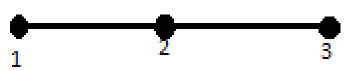
Section 2.5

(1,2)-DOMINATION IN GRAPHS

Definition:

A $(1,2)$-Domination in graphs in a graph G = (V, E) is a set S having the property that for every vertex v in V-S there is atleast one vertex in S at distance one from v and a second vertex in S at distance atmost two from v.The order of the smallest (1,2) Dominating set of G is called (1,2)-Domination number of G and we denote it by $\gamma(1,2)$.

Example:



Here V=$\{1,2,3\}$

S=$\{2,3\}$

V-S=$\{1\}$

Note:-

From the definition of (1,2) Dominating set,we see that a (1,2) Dominating set contains atleast 2 vertices ,(1,2)-Domination number of a graph will always greater than or equal to 2 and (1,2) Dominating set occurs in grph of order atleast 3.

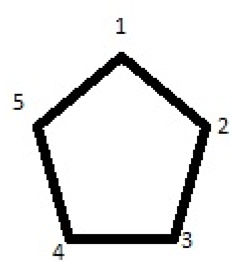
Theorm 2.5.1

All (1,2) Dominating set are dominating sets.

proof:

The result is trivial from the definition of (1,2) Dominating set.But the converse need not be true.

Example:

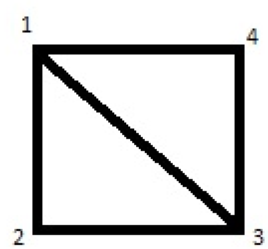


For this $\{1,4\}$ is dominating set.But it is not a (1,2) Dominating set.$\{2,3,4\}$ is a (1,2) Dominating set and Dominating set also.

Theorm2.5.2
(1,2) Domination is not possible in complete graphs.
proof:
In a complete graph , each is adjacent to every other vertices.so we cannot find a (1,2) Dominating set.No vertex can be found at a distance atmost two from ant other vertex.

Note: Let G be a complete graph with n vertices.Then it will have nC2 edges and each vertex is of degree n-1.The minimum number of edges to be deleted so as to become the resulting graph (1,2) Dominating set is n-2 iIf we delete n-2 edges from a complete graph,then in the resulting graph ,we can find a (1,2) Dominating set.

Lemma 2.5.3:
If a graph G with n vertices,has a vertex of degree n-1,we cannot find a (1,2) Dominating set.

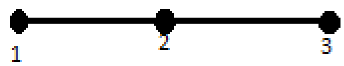


In this graph,we cannot find a (1,2) Dominating set.since each vertex is adjacent to all other vertices.

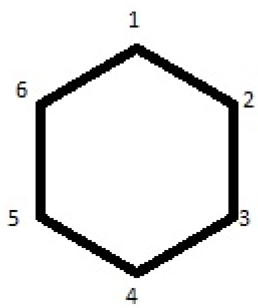
RELATION BETWEEN DOMINATION NUMBER AND (1,2)-DOMINATION NUMBER

In this section we consider different types of graphs and find out their domination number and (1,2)-domination number and check the relation between them.

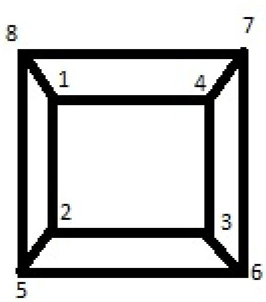
consider the following graphs:



Here $\{2\}$ is dominating set.γ(G)=1.
$\{2,3\}$ is a $\{1,2\}$=2
that is,$\gamma < \gamma_{(1,2)}$

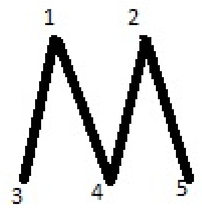


Here $\{1,3,5\}$,$\{2,4,6\}$ are (1,2) Dominating set,γ(G)=3
{1,4,6} is a (1,2) Dominating set.$\gamma_{(1,2)}$=2.
that is γ=$\gamma_{(1,2)}$



Here {1,2,3,4},{5,6,7,8} are dominating.γ(G)=4
$\{1,2,3,4\}$ is a (1,2)- dominating set.$\gamma_{(1,2)}$=4
that is γ=$\gamma_{(1,2)}$

Consider a complete bipartite graph given below.



$\{1, 2\}$ is a dominating set.γ(G)=2

{1,2,4,5}is a (1,2) Dominating set.$\gamma_{(1,2)}$=3.
that is $\gamma<\gamma_{(1,2)}$
In all the above casae,domination number is less than or equal to (1,2)-domination number.
From the above examples we have following theorm.

Theorm 2.5.4
In a graph G,domination number is less than or equal to (1,2)-domination number.
proof:
Let G be a graph and D be its dominating set.Then every vertex in V-D is adjacent to a vertex in D.That is,in D,for every vertex u,there is a vertex which is at distance 1 from u.But it is not necessary that there is a second vertex at distance atmost 2 from u.So if we find a (1,2)-dominating set,it will contain more vertices or atleast equal number of vertices than the dominating set.So the domination number is less than or equal to (1,2)-domination number.

Section 2.6

Other varieties of domination

Paired Domination
Paired Dominating set whose indirect subgraph has a perfect matching from the defnition it requires that there is no isolated vertices.Every Paired Dominating set is Dominating set .Paired Domination number is minimum cardinality of Paired Dominating set and γ_{pr}.

K-Domination
A K-Dominating Set is set of vertices of D such that each vertex in V(G)-D is dominated by atleast k vertices in D for fixed positive integer K.The minimum cardinality of k-dominating set is called k-Dominating number γ_k(G).

Locating Domination
In which we insist that each vertex in V-D has a unique set of vertices D which dominate it.

Distance Domination
In which we insist that each vertex in V-D be within distance k of atleast one vertex in D, for a positive integer K.

Strong Domination
In a graph G=(V,E),a set D of vertices is said to be strong dominating set,for every v in V-D there exist a vertex u in D such that uv∈E(G) and deg u¿=deg v,the subgraph of G.The cardinality of minimum strong dominating set is denoted by γ_s-set.

Distance 2-Domination
A set D is a Distance 2-dominating set if for every vertex u∈V-D,d(u,D) <=2 and is denoted by $\gamma_{<=2}$(G).The Distance 2-Domination number $\gamma_{<=2}$ of G is equals the minimum cardinality of distance 2-dominating set.

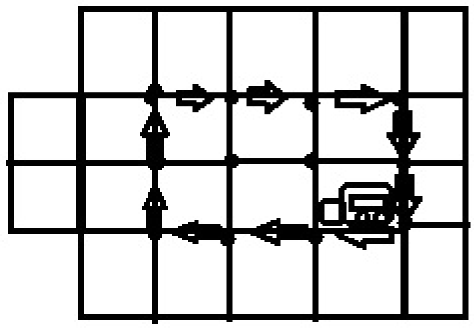
CHAPTER 3

APPLICATION OF DOMINATION IN GRAPH

Domination in graphs has applications to several fields. Domination arises in facility location problems, where the number of facilities (e.g., hospitals, fire stations) is fixed and one attempts to minimize the distance that a person needs to travel to get to the closest facility. A similar problem occurs when the maximum distance to a facility is fixed and one attempts to minimize the number of facilities necessary so that everyone is serviced. Concepts from domination also appear in problems involving finding sets of representatives, in monitoring communication or electrical networks, and in land surveying (e.g., minimizing the number of places a surveyor must stand in order to take height measurements for an entire region).

School bus routing

Most school in the country provide school buses for transporting children to and from school Most also operate under certain rules, one of which usually states that no child shall have to walk farther than, say one quarter km to a bus pickup point. Thus, they must construct a route for each bus that gets within one quarter km of every child in its assigned area. No bus ride can take more than some specified number of minutes, and Limits on the number of children that a bus can carry at any one time. Let us say that the following figure represents a street map of part of a city, where each edge represents one pick up block. The school is located at the large vertex. Let us assume that the school has decided that no child shall have to walk more than two blocks in order to be picked up by a school bus. Construct a route for a school bus that leaves the school, gets within two blocks of every child and returns to the school.



Locating radar stations problem
The problem was discussed by Berge . A number of strategic locations are to be kept under surveillance. The goal is to locate a radar for the surveillance at as few of these locations as possible. How a set of locations in which the radar stations are to be placed can be determined

Nuclear power plants problem
z similar known problem is a nuclear power plants problem. There are various locations and an arc can be drawn from location x to location y if it is possible for a watchman stationed at x to observe a warning light located at y. How many guards are needed to observe all of the warning lights, and where should they be located? At present, domination is considered to be one of the fundamental concepts in graph theory and its various applications to ad hoc networks, biological networks, distributed computing, social networks and web graphs [1, 25, 27, 47] partly explain the increased interest. Such applications usually aim to select a subset of nodes that will provide some definite service such that every node in the network is close to some node in the subset. The following examples show when the concept of domination can be applied in modeling real-life problems.

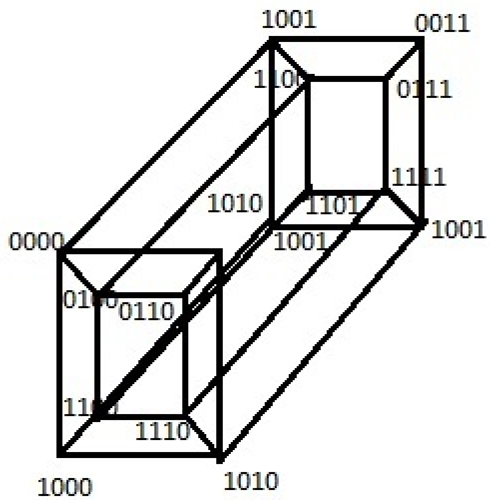
Modeling biological networks
Using graph theory as a modeling tool in biological networks allows the utilization of the most graphical invariants in such a way that it is possible to identify secondary RNA (Ribonucleic acid) motifs numerically. Those graphical invariants are variations of the domination number of a graph.

The results of the research carried out in show that the variations of the domination number can be used for correctly distinguishing among the trees that represent native structures and those that are not likely candidates to represent RNA.

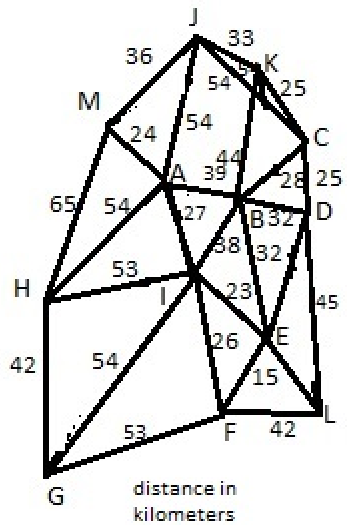
Modeling social networks
Dominating sets can be used in modeling social networks and studying the dynamics of relations among numerous individuals in different domains. A social network is a social structure made of individuals (or groups of individuals), which are connected by one or more specific types of interdependency. The choice of initial sets of target individuals is an important problem in the theory of social networks. In the work of Kelleher and Cozzens, social networks are modeled in terms of graph theory and it was shown that some of these sets can be found by using the properties of dominating sets in graphs.

Facility location problem
The dominating sets in graphs are natural models for facility location problems in operational research. Facility location problems are concerned with the location of one or more facilities in a way that optimizes a certain objective such as minimizing transportation cost, providing equitable service to customers and capturing the largest market share.

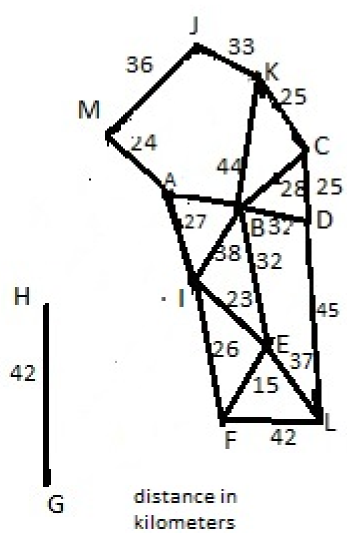
Computer communication network
Consider a computer network modeled by a graph G = (V,E), for which vertices represents computers and edges represent direct links between pairs of computers. Let the vertices in following figure represent an array, or network, of 16 computers, or processors. Each processor to which it is directly connected. Assume that from time to time we need to collect information from all processors. We do this by having each processor route its information to one of a small set of collecting processors (a dominating set). Since this must be done relatively fast, we cannot route this information over too long a path. Thus we identify a small set of processors which are close to all other processors. Let us say that we will tolerate at most a two unit delay between the time a processor sends its information and the time it arrives at a nearby collector. In this case we seek a distance-2 dominating set among the set of all processors. The two shaded vertices form a distance-2 dominating set in the hypercube network in following figure



Radio stations
Suppose that we have a collection of small villages in a remote part of the world. We would like to locate radio stations in some of these villages so that messages can be broadcast to all of the villages in the region. Since each radio station has a limited broadcasting range, we must use several stations to reach all villages. But since radio stations are costly, we want to locate as few as possible which can reach all other villages. Let each village be represented by a vertex. An edge between two villages is labeled with the distance, say in kilometers, between the two villages



Let us assume that a radio station has a broadcast range of fifty kilometers. What is the least number of stations in a set which dominates within distance 50 all other vertices in this graph? A set B,F,H,J of cardinality four is indicated in the following figure 2.



Here we have assumed that a radio station has a broadcast range of only fifty kilometers, we can essentially remove all edges in the graph, which represent a distance of more than fifty kilometers. We need only to find a dominating set in this graph. Notice that if we could afford radio stations which have a broadcast range of seventy kilometers, three radio stations would sufficient.

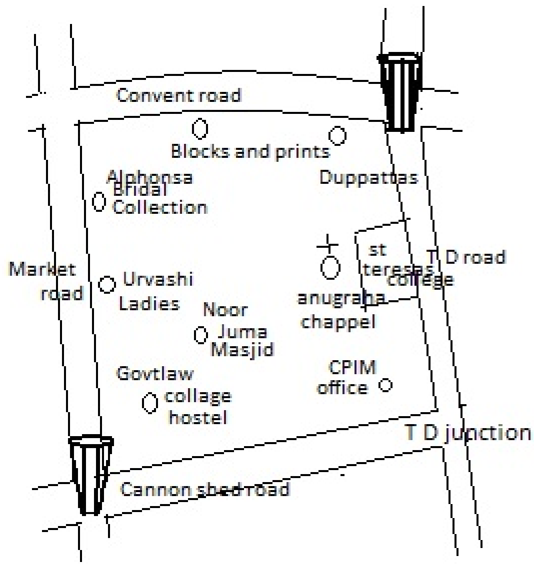
Coding theory

The concept of domination is also applied in coding theory as discussed by Kalbfleisch, Stanton and Horton and Cockayne and Hedetniemi . If one defines a graph, the vertices of which are the n-dimensional vectors with coordinates chosen from 1,..., p, $p > 1$, and two vertices are adjacent if they differ in one coordinate, then the sets of vectors which are (n, p)covering sets, single error correcting codes, or perfect covering sets are all dominating sets of the graph with determined additional properties.

Multiple domination problem

An important role is played by multiple domination. Multiple domination can be used to construct hierarchical overlay networks in peer-to-peer applications for more efficient index searching. The hierarchical overlay networks usually serve as distributed databases for index searching, e.g. in modern file sharing and instant messaging computer network applications. Dominating sets of several kinds are used for balancing efficiency and fault tolerance as well as in the distributed construction of minimum spanning trees. Another good example of direct, important and quickly developing applications of multiple domination in modern computer networks is a wireless sensor network.

? How many waste bins can be placed to reduce pollution in given map(convent junction)?



Two waste bins can be placed in order to reduce pollution.

<u>CONCLUTION</u>

The main aim of this project is to present the importance of graph theoretical ideas in various areas oa science and engineering for domination in graphs theoretical concepts for the research.An overview is presented especially to project the idea of graph theory.so,the graph theory section of each paper is given importance than to other sections.Researches may get some information related to graph theory and its application some ideas related to their research field.

In graph theory,there are many stability parameters such as vertex domination number,independence number etc.The domination number of a graph is a new vulnerability measure that considers the neighbourhood of vertices.From the definition of domination,ever vertex of a graph must be protected by its neigbourhood.In this search,the main ideas is each u,vϵV must be protected and are capable of dominating of both u and v.Also we discussed about total domination,independent domination,(1,2) domination in detail and also discussed varieties of domination and application of domination in different fields.

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