

TB174205C

Reg.No.....

Name.....

**B. Sc. DEGREE (C.B.C.S.S) EXAMINATION, MARCH 2019**  
**(2017 Admissions Regular, 2016 Admissions Improvement/Supplementary & 2015**  
**Admissions Supplementary)**

**SEMESTER IV- CORE COURSE (COMPUTER APPLICATIONS)**  
**CAM4B04TB - VECTOR CALCULUS, THEORY OF EQUATIONS AND GRAPH**  
**THEORY**

**Time: Three Hours**

**Maximum Marks: 80**

**PART A**

**I. Answer all questions. Each question carries 1 mark**

1. Find parametric equation for the line through  $(-2,0,4)$  parallel to  $\mathbf{v} = 2\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$ .
2. Find the gradient field of  $f(x,y,z) = xyz$ .
3. Define symmetric functions of the roots.
4. Define reciprocal equations.
5. Define simple Graph.
6. When a walk is said to be a path?

**(6x1=6)**

**PART B**

**II Answer any seven questions. Each question carries 2 marks**

7. Find parametric equations for the line through  $P(-3,2,-3)$  and  $Q(1,-1,4)$ .
8. Find the point where the line  $x = \frac{8}{3} + 2t$ ,  $y = -2t$ ,  $z = 1+t$  intersects the plane  $3x + 2y + 6z = 6$ .
9. Evaluate  $\int (\cos t \mathbf{i} + \mathbf{j} - 2t\mathbf{k}) dt$ .
10. State fundamental theorem of line integrals.
11. Verify divergence theorem for the field  $\mathbf{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$  over the sphere  $x^2 + y^2 + z^2 = a^2$ .
12. A coil spring lies along the helix  $\mathbf{r}(t) = (\cos 4t)\mathbf{i} + (\sin 4t)\mathbf{j} + t\mathbf{k}$ ,  $0 \leq t \leq 2\pi$ . The spring's density is a constant,  $\delta = 1$ . Find the spring's mass and moment of inertia about the z-axis.
13. If  $\alpha, \beta, \gamma$  are the roots of  $x^3 - x - 1 = 0$ , then find the equation whose roots are  $\frac{1+\alpha}{1-\alpha}, \frac{1+\beta}{1-\beta}, \frac{1+\gamma}{1-\gamma}$ .
14. Solve  $x^4 - 5x^3 + 4x^2 + 8x - 8 = 0$ , given that one of the root is  $1-\sqrt{5}$ .
15. Define  
i) Path ii) Cut vertex iii) Bridge iv) trail
16. Let  $G$  be connected graph. Then prove that  $G$  is a tree if and only if every edge of  $G$  is a bridge.

**(7x2=14)**

## PART C

### III. Answer any five questions. Each question carries 6 marks

17. a) Find the derivative of  $f(x,y) = x^2 \sin 2y$  at the point  $(1, \frac{\pi}{2})$  in the direction of  $v = 3i - 4j$ .
- b) Find an equation for the tangent to the ellipse  $\frac{x^2}{4} + y^2 = 2$ .
18. a) Estimate how much the value of  $f(x,y,z) = y \sin x + 2yz$  will change if the point  $P(x,y,z)$  moves 0.1 unit from  $P_0(0,1,0)$  straight toward  $P_1(2,2,-2)$ .
- b) Also find plane tangent to the surface  $z = 1 - \frac{1}{10}(x^2 + 4y^2)$  at  $(1, 1, \frac{1}{2})$ .
19. a) A fluid's velocity field  $F = xi + zj + yk$ . Find the flow along the helix  $r(t) = (\cos t) i + (\sin t)j + tk, 0 \leq t \leq \frac{\pi}{2}$ .
- b) Also find the parametrization of the cylinder  $x^2 + (y - 3)^2 = 9, 0 \leq z \leq 5$ .
20. a) State Green's theorem (normal form and tangential form)
- b) Using Green's theorem, calculate the outward flux of the field  $F(x,y) = x^2i + xyj$  across the square bounded by the lines  $x = 0, y = 0, x = a$ , and  $y = a$  ( $a > 0$ )
21. a) prove that every equation of the  $n$ th degree has exactly  $n$  roots.
- b) If  $\alpha, \beta, \gamma$  are the roots of the function  $f(x) = x^3 + P_1x^2 + P_2x + P_3 = 0$ , find the equation whose roots are  $\alpha^3, \beta^3, \gamma^3$ .
22. Solve  $x^3 - 27x + 54 = 0$ , using Cardan's method.
23. A connected graph  $G$  with  $n$  vertices has at least  $n-1$  edges. Prove.
24. Prove : a graph  $G$  is connected if and only if it has a spanning tree.

(5x6=30)

## PART D

### IV. Answer any two questions. Each question carries 15 marks

25. a) Prove that the curvature of a circle of radius  $a$  is  $\frac{1}{a}$ .
- b) Find and graph the osculating circle of the parabola  $y = x^2$  at the origin.
26. a) Find the net outward flux of the field  $F = \frac{xi+yj+zk}{\rho^3}, \rho = \sqrt{x^2+y^2+z^2}$  across the boundary of the region  $D : 0 < a^2 \leq x^2+y^2+z^2 \leq b^2$ .
- b) Verify the circulation form of green's theorem on the annular ring  $R : h^2 \leq x^2 + y^2 \leq 1, 0 < h < 1$ , if  $M = \frac{-y}{x^2+y^2}, N = \frac{x}{x^2+y^2}$ .
27. Using Ferrari's method, solve  $x^4 - 10x^2 - 20x - 16 = 0$ .
28. a) Let  $e$  be an edge of the graph  $G$  and, let  $G-e$  be the subgraph obtained by deleting  $e$ . Then prove that  $\omega(G) \leq \omega(G - e) \leq \omega(G) + 1$ .
- b) Prove : An edge  $e$  of a graph  $G$  is a bridge if and only if  $e$  is not part of any cycle in  $G$ .

(2x15=30)