

B. Sc. DEGREE (C.B.C.S.S.) EXAMINATION, APRIL 2018
(2014 Admission Supplementary)
SEMESTER II - COMPLEMENTARY COURSE (STATISTICS)
STA2TRV – THEORY OF RANDOM VARIABLES
(Complementary for Mathematics, Physics and Core for Computer Applications)

Time: Three Hours

Maximum Marks: 80

Use of Scientific calculators and Statistical tables are permitted.

PART A**I. Answer all questions. Each question carries 1 mark.**

1. State the properties of p.d.f. of a random variable.
2. If the p.d.f. of a random variable X is $f(x) = kx^2$; $0 < x < 1$, find k.
3. Define distribution function of a random variable X.
4. For a discrete random variable X, show that $E(aX + b) = aE(X) + b$.
5. If the moment generating function of a random variable X is $(1 - t)^{-1}$, find $E(X)$.
6. If $f(x) = \frac{1}{2}$, $-1 < x < 1$ is the p.d.f. of a random variable X, find $\phi_X(t)$.
7. Give the formula for Bowley's co-efficient of skewness.
8. What are the limits within which the correlation co-efficient of a bivariate data must lie?
9. What is a Scatter diagram?
10. Give the relation between regression co-efficients and correlation co-efficient of a bivariate data.

(10x1=10)**PART B****II. Answer any eight questions. Each question carries 2 marks.**

11. Distinguish between Discrete and Continuous random variables.
12. Can the following be a probability density function?
 $g(x) = \frac{1}{2}$ for $x = 1$
 $= \frac{2}{3}$ for $x = 0$
 $= \frac{1}{4}$ for $x = 2$ and 0 elsewhere.
13. Find k if $f(x) = kx(1-x)$; $0 \leq x \leq 1$ and 0 elsewhere is a p.d.f of a continuous random variable.
14. For any two independent random variables X and Y, show that $E(XY) = E(X)E(Y)$.
15. Define characteristic function of a random variable and state its important properties.
16. The joint p.d.f. of a bivariate random variable (X, Y) is
 $f(x, y) = x + y$; $0 < x < 1$, $0 < y < 1$, obtain the marginal p.d.f. of Y.
17. What is Sheppard's correction? Write down the Sheppard's correction for the first four moments.
18. The first three raw moments of a distribution are 1, 3 and 5 respectively. Find S_1 .
19. What are the normal equations to fit a curve of the form $y = a + bx$ to a given bivariate data?

20. Find Spearman's rank correlation co-efficient from the following data.
- | | | | | | | | | | | |
|--------------|---|---|---|---|---|----|---|----|---|---|
| Individuals: | A | B | C | D | E | F | G | H | I | J |
| Rank before: | 1 | 6 | 3 | 9 | 5 | 2 | 7 | 10 | 8 | 4 |
| Rank after: | 6 | 8 | 3 | 2 | 7 | 10 | 5 | 9 | 4 | 1 |
21. Show that the regression lines of a bivariate data intersect at (\bar{x}, \bar{y}) .
22. If the two regression equations of a bivariate data are $14x + 12y - 3 = 0$ and $12x + 21y + 10 = 0$, find co-efficient of correlation between x and y.

(8x2 = 16)

PART C

III. Answer any six questions. Each question carries 4 marks.

23. Write down the probability distribution of X and Y, where X denotes the sum of the numbers obtained and Y denotes the maximum of the numbers obtained.
24. A random variable X has p.d.f. $f(x) = \frac{1}{4}$; $-2 < x < 2$. Obtain the p.d.f. of $Y = X^2$.
25. Find the moment generating function of a random variable X whose p.d.f is $f(x) = a^x b$; $x = 0, 1, 2, \dots$, where $a + b = 1$. Hence find $V(X)$.
26. State and prove Cauchy-Schwartz inequality.
27. If X and Y are any two random variables, show that $E(E(X|Y)) = E(X)$.
28. For a distribution mean = 3, variance = 4, $S_1 = +1$ and $S_2 = 2$. Obtain the first four moments about zero.
29. Fit an equation of the form $y = ax + b$ to the following data
- | | | | | | | |
|----|---|----|----|----|----|----|
| x: | 1 | 3 | 5 | 7 | 8 | 10 |
| y: | 8 | 12 | 15 | 17 | 18 | 20 |
30. Show that Karl Pearson's co-efficient of correlation is independent of change of origin and scale.
31. Derive the expression for the angle between the two regression lines of a bivariate data.

(6x4 = 24)

PART D

IV. Answer any two questions. Each question carries 15 marks.

32. Define conditional expectation and conditional variance. If $f(x,y) = x+y$; $0 < x < 1$, $0 < y < 1$ is the joint p.d.f. of (X,Y), find correlation between X and Y.
33. What is Skewness? Obtain the moment measure of Skewness from the following data
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|-----------|---|----|----|----|----|----|----|
| Variable | 5 | 10 | 15 | 20 | 25 | 30 | 35 |
| Frequency | 4 | 38 | 65 | 90 | 70 | 42 | 6 |
34. Find Karl Pearson's co-efficient of correlation and Spearman's rank correlation co-efficient from the following data
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|----|-----|-----|-----|----|----|-----|----|-----|----|-----|
| X: | 115 | 109 | 112 | 87 | 98 | 120 | 98 | 100 | 98 | 118 |
| Y: | 75 | 73 | 85 | 70 | 76 | 82 | 65 | 73 | 68 | 80 |
35. Given the following observations on a bivariate data (x,y), find the most probable value of x when $y = 72$
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|----|----|----|----|----|----|----|----|----|----|----|
| x: | 59 | 65 | 45 | 52 | 60 | 62 | 70 | 55 | 45 | 49 |
| y: | 75 | 70 | 55 | 65 | 60 | 69 | 80 | 65 | 59 | 61 |

(2x15 = 30)