B.Sc. DEGREE (C.B.C.S.S) EXAMINATION, APRIL 2018
(2014 Admission Supplementary)
SEMESTER II – COMPLEMENTARY COURSE (MATHEMATICS)
MAT2ICM – INTEGRAL CALCULUS AND MATRICES
(For Chemistry & Physics)

Time: 3 Hours   **Maximum marks: 80**

PART A

I. Answer all questions. Each question carries 1mark.

1. Find $\int_0^\pi \cos x \, dx$.

2. Let $\int_0^2 f(z)dz = 3$. Find $\int_2^0 f(z)dz$.

3. Find $\int_0^2 \sqrt{y+1} \, dy$.

4. Let f be an odd function. Find $\int_{-1}^1 f(x)dx$.

5. Find $\int_0^1 2x\sqrt{x^2+1} \, dx$.

6. Define rank of a matrix.

7. Define singular matrix.

8. State Cayley Hamilton theorem.

9. Find $\begin{vmatrix} 1 & 2 & 3 \\ -1 & -2 & -3 \\ 2 & 4 & 6 \end{vmatrix}$.

10. Find $\int_0^1 \int_0^1 (x+y)dy \, dx$.

                                                     $(10 \times 1 = 10)$

PART B

II. Answer any eight questions. Each question carries 2 marks.

11. Find $\int_0^\pi \cot\theta \csc^2\theta \, d\theta$.

12. State Max Min inequality.

13. Show that the value of $\int_0^1 \sin x^2 \, dx$ cannot possibly be 2

14. Find the area of the region bounded by the lines $y = x$ and $y = 2$.

15. Find the length of the curve : $x = \cos t, y = t + \sin t; 0 \leq t \leq \pi$.

16. Find the volume of the solid generated by revolving the region between the X-axis and the curve $y = \sqrt{x}; 0 \leq x \leq 4$ about X-axis.

17. Find the area of the region between the X axis and the graph of
    $f(x) = x^3 - x^2 - 2x, -1 \leq x \leq 2$.

18. Find $\int_0^1 \int_0^1 \int_0^1 (x^2 + y^2 + z^2)dz \, dy \, dx$.

19. State the first form of Fubini's theorem.

20. Find the average value of $f(x,y) = x\cos xy$ over the rectangle;
    $R: 0 \leq x \leq \pi, 0 \leq y \leq \pi$

21. Test for consistency:

$$4x + 3y + 7z = 14$$
$$8x + 5y - 2z = 13$$

22. Find the characteristic polynomial of $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & 5 \\ 0 & 0 & 4 \end{bmatrix}$.

<div align="right">(8 × 2 = 16 marks)</div>

PART C

III. Answer any six questions. Each question carries 4 marks.

23. Give an example of a non integrable function on [0, 1].

24. Evaluate $\int_0^1 \int_0^\pi \int_0^1 y \sin z \, dx \, dy \, dz$.

25. Evaluate $\int \frac{1}{1-x^2} \, dx$.

26. Find the length of the circle of radius r defined by $x = r \cos t$ and $y = r \sin t$; $0 \ \ t \le 2\pi$.

27. Change $\int_0^1 \int_0^{\sqrt{1-x^2}} dy \, dx$ into polar integral and evaluate it.

28. Find the area enclosed by the cardioid $r = a(1 + \cos \theta)$.

29. Reduce $\begin{bmatrix} 1 & 0 & 2 & 3 \\ 2 & 1 & 0 & 1 \\ 4 & 1 & 4 & 7 \end{bmatrix}$ to the normal form.

30. Show that the eigen values of the diagonal matrix are its diagonal elements.

31. Find the eigen vectors of $A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$.

<div align="right">(6 × 4 = 24)</div>

PART D

IV. Answer any two questions. Each one carries 15 marks.

32. i) Find the area between the graph of $y = x^3 - 4x$ and X-axis over $[-2,2]$

   ii) Sketch the region bounded by the coordinate axes and the line $x + y = 2$. Also find the area of the region.

33. i) A pyramid 3m high has a square base that is 3m on a side. The cross-section of the pyramid perpendicular to the altitude x m down from the vertex is a square x m on a side. Find the volume of the pyramid.

   ii) Find the volume of the region bounded by $y = 2x, y = 0, x = 1$ about the line $x = 1$.

34. Evaluate $\int_0^{\sqrt{2}} \int_0^{3y} \int_{x^2+3y^2}^{8-x^2-y^2} dz \, dy \, dx$

35. Solve using Cramer's rule:

$$2x + y + 5z + w = 5$$
$$x + y - 3z - 4w = -1$$
$$3x + 6y - 2z + w = 8$$
$$2x + 2y + 2z - 3w = 2$$

<div align="right">(2 × 15 = 30)</div>