TB142330C **Reg. No:...............................**

Name:..................................

B. Sc. DEGREE (C.B.C.S.S) EXAMINATION, APRIL 2018

(2014 Admission Supplementary)

SEMESTER II - CORE COURSE (MATHEMATICS)

MAT2AGTM - ANALYTIC GEOMETRY, TRIGONOMETRY AND MATRICES

Time: Three Hours **Maximum Marks: 80**

PART A

I. Answer all the questions. Each question carries 1 mark.

1. What is the polar of the focus with respect to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.
2. Write the equation of the tangent at‘t’ to the parabola $y^2 = 4ax$.
3. Define orthoptic locus of a conic.
4. Write the standard equation of a polar equation of the conic.
5. Define conjugate diameter of the hyperbola$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$.
6. Prove that $\cos h^2 x - sinh^2x = 1$.
7. Define $\sin hx$.
8. Define rank of a matrix.
9. Define Equivalent matrices.
10. If the rank of a matrix is 4, what is the rank of its transpose?

(10x1=10)

PART B

II. Answer any eight questions. Each question carries 2 marks.

11. The chord joining 2 points t_1 and t_2 on the parabola $y^2 = 4ax$, passes through the focus. Prove that $t_1 t_2 = -1$.
12. If e_1 and e_2 are the eccentries of a hyperbola and its conjugate, show that $\frac{1}{{e_1}^2}+\frac{1}{{e_2}^2}=1$.
13. Show that the sum of the squares of 2 conjugate semi-diameters of an ellipse is constant.
14. If the normal at $(a{t_1}^2, 2at_1)$ to the parabola $y^2 = 4ax$ meets the curve again at $(a{t_2}^2, 2at_2)$. Shiw that $t_1 = -t_1 - \frac{2}{t_1}$.
15. Show that 3 normals can be drawn from a given point to a parabola $y^2 = 4ax$.
16. Find the polar equation of a straight line, whose perpendicular distance from the pole is p, perpendicular making angle α with the initial line.
17. Show that $\log(\sqrt{3}+i) = \log 2 + i\frac{\pi}{6}$.
18. If $\tan\frac{\theta}{2} = \tan h\frac{u}{2}$. Show that $\sin \text{hu} = \tan\theta$.
19. Find the real and imaginary part of $\sin(\alpha - i\beta)$.
20. Write the matrix form of the system equation

$$7x - 4y + 3z = 1$$
$$8x + 3y - 5z = 6$$
$$-9x + y + 10z = 2.$$

21. Find the rank of the matrix $A = \begin{pmatrix}1 & 4\\ 5 & 6\end{pmatrix}$.
22. Define characteristic equation of a matrix.

(8x2=16)

PART C

III. Answer any six questions. Each question carries 4 marks.

23. Derive the equation of a rectangular hyperbola referred to its asymptotes as the axes.

24. Show that the locus of the poles of normal chords of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is the curve $y^2a^6 - x^2b^6 = (a^2 + b^2)^2x^2y^2$.

25. Prove that the tangents at the extremities of any focal chord of a parabola intersect at the right angles on the directrix.

26. If PSP' is a focal chord of a conic, S is a focus and SL is the semi latus rectum. Prove that $\frac{2}{SL} = \frac{1}{SP} + \frac{1}{SP'}$.

27. Factorize $x^8 + 1$ into real factors.

28. If $\tan(A + iB) = x + iy$. Prove that $x^2 + y^2 + 2x\cot 2A = 1$.

29. Using normal form, find the rank of a matrix $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{bmatrix}$.

30. Verify Cayley Hamilton theorem for the matrix A and also find A^{-1} , where A= $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

31. Solve the following system of equation by Cramer's rule.

$$2x_1 + x_2 + 5x_3 + x_4 = 5$$
$$x_1 + x_2 - 3x_3 - 4x_4 = -1$$
$$3x_1 + 6x_2 - 2x_3 + x_4 = 8$$
$$2x_1 + 2x_2 + 2x_3 - 3x_4 = 2.$$

(6x4=24)

PART D

IV. Answer any two questions. Each question carries 15 marks.

32. A rectangular hyperbola whose centre C is cut by any circle of radius r in the 4 points P,Q,R,S. Prove that $CP^2 + CQ^2 + CR^2 + CS^2 = 4r^2$.

33. Tangents are drawn to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ from points on the circle $x^2 + y^2 = c^2$. Show that the middle point of the points of contact lies on the curve $(\frac{x^2}{a^2} + \frac{y^2}{b^2})^2 = \frac{x^2+y^2}{c^2}$.

34. Sum to infinity $\cos x \sin x + \frac{1}{2!}\cos^2 x \sin 2x + \frac{1}{3!}\cos^3 x \sin 3x + \cdots$

35. Solve by matrix method

$$x + 2y + z = 2$$
$$3x + y - 2z = 1$$
$$4x - 3y - z = 3$$
$$2x + 4y + 2z = 4.$$

(2x15=30)