

TB156495A

Reg. No.....

Name.....

**B. Sc. DEGREE (C.B.C.S.S) EXAMINATION, MARCH 2018**  
**(2015 Admission Regular)**  
**SEMESTER VI – CORE COURSE (COMPUTER APPLICATIONS)**  
**CAM6B07TB - REAL ANALYSIS II**

**Time: Three Hours**

**Maximum Marks: 80**

**PART A**

**I. Answer all questions. Each question carries 1 mark.**

1. State Cauchy's General Principle of Convergence.
2. Define Absolute Convergence.
3. Define continuity at an internal point 'c' of a function f.
4. For any two partitions  $P_1, P_2$ , Show that  $L(P_1, f) \leq U(P_2, f)$ .
5. Define uniform convergence of a sequence of functions.
6. Show that  $\sum \frac{\cos nx}{n^2}$  is uniformly convergent for all values of  $x$ , where  $p > 1$ .

**(6X1=6)**

**PART B**

**II. Answer any seven questions. Each question carries 2 marks.**

7. State and Prove Geometric Series Test.
8. Show that the series  $\sum (\log n)^{-p}$  diverges for  $p > 0$ .
9. Prove that a series  $\sum u_n$  of positive terms converges if  $\lim_{n \rightarrow \infty} (u_n)^{\frac{1}{n}}$  exists and is less than 1.
10. If a function f is continuous on a closed interval [a,b] then prove that it attains its bounds at least ones in [a,b].
11. Show that the function  $f(x) = x^2$  is uniformly continuous on [-1,1]
12. Show that the function  $f(x) = \frac{1}{1+|x|}$  for real x is continuous and bounded and attains its supreme for  $x=0$  but does not attain the infimum.
13. Show that the function f defined by  
$$f(x) = \begin{cases} 0, & 'x \text{ is rational}' \\ 1, & 'x \text{ is irrational}' \end{cases}$$
is not integrable on any interval.
14. Show that if  $f$  is bounded and integrable on  $[a, b]$  and  $k$  is a number such that  
$$|f(x)| \leq k \quad \forall x \in [a, b] \text{ then } \left| \int_a^b f(x) dx \right| \leq k|b - a|.$$
15. State and prove Weierstrass M test.

16. Show that  $\{f_n\}$  where  $f_n(x) = \frac{1}{x+n}$  is uniformly convergent in  $[0, b], b > 0$ .

(7X2=14)

### PART C

#### III. Answer any five questions. Each question carries 6 marks.

17. State and Prove D'Alembert's Ratio Test.
18. Test the convergence of the series:  
$$\frac{\alpha}{s} + \frac{1+\alpha}{1+s} + \frac{(1+\alpha)(2+\alpha)}{(1+\beta)(2+\beta)} + \frac{(1+\alpha)(2+\alpha)(3+\alpha)}{(1+\beta)(2+\beta)(3+\beta)} + \dots$$
19. Show that the function  $f$  defined by  
$$f(x) = \begin{cases} 1, & \text{if } x \text{ is rational} \\ 0, & \text{if } x \text{ is irrational} \end{cases}$$
 is discontinuous at every point.
20. State that if a function  $f$  is continuous on a  $[a, b]$ , then it attains bounds at least once in  $[a, b]$ .
21. State and Prove a characterization for the integrability of a bounded function.
22. Without doing direct integration show that  $\int_1^2 f dx = \frac{11}{2}$  where  $f(x) = 3x + 1$ .
23. Consider the series  $\sum_{n=0}^{\infty} f_n$  where  $f_n(x)$  is defined as  $\frac{x^2}{(1+x^2)^n}, x \in \mathbf{R}$ . Find its sum function.
24. Show that  $\{f_n(x)\}$  where  $f_n(x) = x^n$  is uniformly convergent on  $[0, k], k < 1$ .

(5X6=30)

### PART D

#### IV. Answer any two questions. Each question carries 15 marks.

25. State and Prove Gauss Test.
26. Show that if the function  $f$  is continuous on a  $(a, b)$  and  $f(a)$  and  $f(b)$  are the opposite signs, then there exists at least one point  $\tau \in (a, b)$  such that  $f(\tau) = 0$ .
27. If a function  $f$  is bounded and integrable on  $[a, b]$ . Prove that the function  $F$  defined as  
$$F(x) = \int_a^x f(t) dt, a \leq x \leq b$$
 is continuous on  $[a, b]$  and furthermore if  $f$  is continuous at a point  $c$  of  $[a, b]$ , then  $F$  is derivable at  $c$  and  $F'(c) = f(c)$ .
28. State and Prove Cauchy's criterion for uniform convergence for a sequence of functions  $\{f_n\}$ .

(2X15=30)