Reg. No.................................

Name....................................

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MARCH 2016

Sixth Semester

Choice Based Course—OPERATIONS RESEARCH

(For B.Sc. Mathematics Model I)

[2013 Admissions]

Time : Three Hours   Maximum Marks : 80

Part A

Answer **all** *questions.*
Each question carries 1 *mark.*

1. Let V be set of all polynomials in x of degree nor less. Show that V is a Vector space.
2. What do you mean by dimension of a Vector space ?
3. What are open and closed sets ?
4. Define extreme point of a convex set K.
5. What is a basic feasible solution of an LPP ?
6. What is Unbalanced Transportation Problem ?
7. Why the Transportation algorithm is not preferred in solving an Assignment Problem ?
8. When do you say that a queueing system has attained a steady state ?
9. What is meant by the statement "the traffic intensity is 0.20" related to a queueing system ?
10. Write a note on Behaviour of customers in a queue.

(10 × 1 = 10)

Part B

Answer any **eight** *questions.*
Each question carries 2 *marks.*

11. Define subspace of a Vector space. Give examples.
12. Check whether the vectors $x_1 = (1, 0, 0)'$, $x_2 = (1, 1, 0)'$ and $x_3 = (1, 1, 1)'$ are linearly independent.

    If so, express $x = (5, 6, 7)'$ as a linear combination of x_1, x_2 and x_3.

13. Show that $\| x \| = | x_1 | + | x_2 | + \ldots\ldots + | x_n |$ statistics all the conditions of a norm of n-vector x.

14. Show that the convex hull of S is the set of all convex linear combinations of points in S.

15. Prove that dual of the dual is primal.

16. What are artificial variables ? What are its uses ?

17. What is a loop in a Transportation table ? Give examples.

18. Write the Mathematical Model of an Assignment Problem. How this problem is related to Transportation problem ?

19. Solve the dual of the following problem graphically :

$$\begin{aligned} &\text{Minimize } x_1 + x_2 \\ &\text{subjected to } 2x_1 + x_2 \geq 8, \\ &\qquad 3x_1 + 7x_2 \geq 21, \\ &\qquad x_1, x_2 \geq 0. \end{aligned}$$

20. Explain Vogel's approximation method.

21. What are the axioms used about the number of arrivals and departures during an interval of item in a queueing system ?

22. What do you mean by queue discipline ? Discuss any *two* of them.

(8 × 2 = 16)

Part C

Answer any **six** *questions.*
Each question carries 4 *marks.*

23. State and prove Cauchy-Schwarz inequality.

24. Prove that any inter section of closed sets is again closed.

25. Prove that the value of the primal objective function for any feasible solution of the primal is not less than the value of the dual objective function for any feasible solution of the dual.

26. A person has the option of investing Rs. 10,000 in two plans A and B, Plan A guarantees a return of 50 paisa on each rupee invested after a period of three years, and Plan B guarantees that each rupee invested will earn one and a half rupees after six years. How should the person invest his money to maximize his earnings in a period of six years, if he is not willing to invest more than 60 % in B ?

27. Solve using Dual simplex method :

$$\text{Minimize } Z\ 300x + 110y$$
$$\text{subjected to } 30x + 5y \geq 6,$$
$$20x + 10y \geq 8,$$
$$x \geq 0, y \geq 0.$$

28. Solve the following Transportation Problem :

		Destinations			
		D_1	D_2	D_3	Availability
	O_1	10	6	12	60
Origins	O_2	11	9	11	130
	O_3	8	7	10	65
Demand		125	70	100	

29. A company has four machines used for three jobs. Each job can be assigned to one and only one machine. The time taken to perform each job by each machine is given below :

		Machine			
		M_1	M_2	M_3	M_4
	J_1	23	29	33	37
Job	J_2	13	18	22	23
	J_3	15	20	24	27

Find the optimal assignment to minimize the total processing time.

30. Distinguish between static and dynamic arrival process.

31. If the number of arrivals n, intime t follows Poisson distribution find the distribution of the inter arrival times.

(6 × 4 = 24)

Part D

Answer any **two** *questions.*
Each question carries 15 *marks.*

32. (a) If the set K is non-empty, closed, convex and bounded from below (or above) then prove that it has atleast one vertex.

(b) Prove the every point of the convex hull of S ([S]) can be expressed as a convex linear combination of atmost $(n + 1)$ points of $S \subseteq E_n$.

33. (a) Prove that a vertex of a set of feasible solutions is a basic feasible solution.

(b) Prove that the negative of the simplex multipliers for the optimal solution of the primal are the values of the variables for the optimal solution of the dual.

34. (a) Prove that the transportation problem has a triangular basis.

(b) Describe an algorithm to solve an Assignment Problem.

35. If the arrivals are completely random. Show that the probability distribution of numbers of arrivals in a fixed time interval follows a Poisson distribution.

$(2 \times 15 = 30)$