E 1550 (Pages : 3) Reg. No..............................

Name..............................

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MARCH 2016

Sixth Semester

Core Course—LINEAR ALGEBRA AND METRIC SPACES

(2013 Admissions)

Time : Three Hours Maximum : 80 Marks

Part A

*Answer **all** questions each in a sentence* **or** *two.*
Each question carries 1 *mark.*

1. Define zero vector in a vector space.
2. Give an example of a linearly independent set in $\mathbb{R}^2$.
3. Define dimension of a vector space.
4. Give an example of an onto function.
5. Define nullity of a linear transformation.
6. Define the linear transformation 'projection'.
7. Give an example of a bounded function.
8. Show that empty set is an open set in any metric space.
9. Define closed set.
10. Give an example of a complete metric space.

(10 × 1 = 10)

Part B (Short Notes)

Answer any **eight** *questions.*
Each question carries 2 *marks.*

11. Show that the additive inverse of a vector in a vector space V is unique.
12. Check whether the set of all 3 × 3 real upper triangular matrices under standard matrix addition and scalar multiplication is a vector space.
13. Show that a subset of a vector space V consisting of the single vector u is linearly dependent if and only if $u = 0$.
14. Prove or disprove that the function $T:\mathbb{R}^2 \to \mathbb{R}^2$ given by $T[a,b]=[a,1]$ is linear.
15. If $T : V \to W$ is a linear transformation, then prove that T (0) = 0.

Turn over

16. Let a linear transformation $T:V \to W$ have the property that the dimension of V equals the dimension of W. Then prove that T is one-to-one if and only if T is onto.

17. Let X be metric space. If $\{x\}$ is a subset of X consisting of a single point, show that its complement $\{x\}'$ is open.

18. Let X be a metric space. Then prove that any finite union of closed sets in X is closed.

19. Let X be an arbitrary metric space, and let A be a subset of X. If $A = \overline{A}$ then prove that A is closed.

20. Show that the boundary of a set is closed.

21. Let X be a metric space with metric d. If $\{x_n\}$ and $\{y_n\}$ are sequences in X such that $x_n \to x$ and $y_n \to y$, show that $d(x_n, x) \to d(x, y)$.

22. Define uniformly continuous function and give an example.

(8 × 2 = 16)

Part C

Answer any **six** *questions.*
Each question carries 4 marks.

23. Show that the span of the set of vectors $S = \{v_1, v_2, \ldots, v_n\}$ in a vector space V is a subspace of V.

24. Show that every basis for a finite dimensional vector space must contain the same number of vectors.

25. Find a basis for the span of the vectors in $S = \{t^2 + t, t + 1, t^2 + 1, 1\}$.

26. Prove that a matrix A is similar to a matrix B then B is similar to A.

27. Show that the image of a linear transformation $T:V \to W$ is a subspace of W.

28. Prove that a linear transformation $T:V \to W$ is one to one if and only if the image of every linearly independent set of vectors in V is a linearly independent set of vectors in W.

29. Let X be a metric space. Prove that a subset G of X is open if it is a union of open spheres.

30. Define Cantor set and explain its construction.

31. Let X be a complete metric space, and let Y be a subspace of X. Then show that if Y is complete then it is closed.

(6 × 4 = 24)

Part D (Essays)

Answer any **two** *questions.*
Each question carries 15 *marks.*

32. If $S = \{v_1, v_2, \ldots\ldots, v_n\}$ is a basis for a vector space V, then show that any set containing more then n vectors is linearly dependent. Also determine the dimension of P^n.

33. Give an example of a linear transformation such that its Kernel contains only one element.

    Show that a linear transformation $T : V \to W$ is one-to-one if and only if the kernel of T contains just the zero vector.

34. Let X be a metric space. Show that a subset F of X is closed if and only if its complement F' is open.

35. Let X and Y be metric spaces and f a mapping of X into Y. Then show that f is continuous at x_0 if and only if $x_n \to x_0 \Rightarrow f(x_n) \to f(x_0)$.

(2 × 15 = 30)