E 1246 (Pages : 3) Reg. No..................................

Name......................................

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MARCH 2016

Sixth Semester

Core Course—REAL ANALYSIS

(For B.Sc. Mathematics Model I and Model II and B.Sc. Computer Applications)

[Prior to 2013 Admissions]

Time : Three Hours Maximum Weight : 25

Part A

Answer **all** *the questions.*

Each bunch of **four** *questions carries a weight of 1.*

I. 1 State Cauchy's root test.

2 What is the necessary condition for convergence of a positive term series ?

3 Is the series $1 - 1 + 1 - 1 \ldots.$ converges.

4 State limit form of comparison test for positive term series.

II. 5 Define a geometric series.

6 Test the convergence of the series $\Sigma \dfrac{1}{n^1 + \dfrac{1}{n}}$.

7 Define discontinuity of first kind for a function f at a point c.

8 Define uniform continuity.

III. 9 Is the function $f(x) = \dfrac{1}{x}$ continuous on the set $A = \{x \in R : x > 0\}$.

10 Give an example of a function which is discontinuous at every point of R.

11 Define $\int_a^{-b} f\, dx$, where f is a bounded real function on $[a, b]$.

12 State Darboux's theorem.

IV. 13 State Fundamental theorem of Calculus.

14 State Cauchy's criterion for uniform convergence.

15 State Dirichlet's test for uniform convergence.

16 What condition on r enables $\Sigma r^n \cos n\theta$ to converge uniformly for all real values of θ.

$(4 \times 1 = 4)$

Turn over

Part B

Answer any **five** *questions.*
Each question carries a weight of 1.

17 Show that the series $\sum_{n=2}^{\infty}\left(\frac{1}{(\log n)^{P}}\right)$ diverges for $P > 0$.

18 Examine the convergence of the series $\frac{1}{3}+\frac{1.2}{3.5}+\frac{1.2.3}{3.5.7}+\ldots$.

19 Test the convergence of the series whose nth term is $\frac{n^{n^2}}{(n+1)n^2}$.

20 Examine the function

$$f(x)=\begin{cases} x\sin\frac{1}{x} & \text{when } x\neq 0 \\ 0 & \text{when } x=0 \end{cases}$$

for continuity at the origin.

21 Discuss the continuity of $f(x)=x-[x], x\geq 0$ at $x = 3$.

22 Show that the function $f(x)=\begin{cases}0, x \text{ rational}\\ 1, x \text{ irrational}\end{cases}$ is not integrable on any interval.

23 Prove that if f is monotonic on $[a, b]$, then it is integrable on $[a, b]$.

24 Test for uniform convergence, the series $\frac{2x}{1+x^2}+\frac{4x^3}{1+x^4}+\frac{8x^7}{1+x^8}+\ldots, -\frac{1}{2}\leq x=\frac{1}{2}$.

(5 × 1 = 5)

Part C

Answer any **four** *questions.*
Each question carries a weight of 2.

25 Test for convergence the series

$$\Sigma\frac{1^2.3^2\ldots\ldots(2n-1)^2}{2^2.4^2\ldots\ldots(2n)^2}x^{n-1}.$$

26 Show that every absolutely convergent series is convergent.

27 If f is continuous on $[a, b]$ and $f(a)$ and $f(b)$ are of opposite signs, show that there exists at least one point $\alpha\in[a, b]$ such that $f(\alpha) = 0$.

28 If f_1 and f_2 are two bounded and integrable functions on $[a, b]$, show that $f_1 + f_2$ is also integrable on $[a, b]$ and $\int_a^b f_1\,dx+\int_a^b f_2\,dx=\int_a^b f\,dx$.

29 If f is bounded and integrable on $[a, b]$, prove that $|f|$ is also bounded and integrable and

$$\left|\int_a^b f dx\right| \leq \int_a^b |f|\, dx.$$

30 Show that the sequence $\{fu\}$, where $fu\ (x) = x^n$ is uniformly convergent on $[o, k]$, $k < 1$ and only pointwise convergent on $[0, 1]$.

$(4 \times 2 = 8)$

Part D

Answer any **two** *questions.*
Each question carries a weight of 4.

31 State and prove Leibnitz test.

32 Show that a function which is continuous on a closed bounded interval is uniformly continuous on that interval.

33 Prove that if f is bounded and integrable on $[a, b]$, then it is integrable on $[a, c]$ and $[c, b]$, where c is a point in $[a, b]$. Also prove that $\int_a^b f\, dx = \int_a^c f\, dx + \int_c^b f\, dx$.

$(2 \times 4 = 8)$