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Reg. No.....

Name.....

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MARCH 2016

Sixth Semester

Core Course—REAL ANALYSIS

(For B.Sc. Mathematics Model I and Model II and B.Sc. Computer Applications)

[2013 Admissions]

Time : Three Hours

Maximum Marks : 80

Part A

Answer all questions.

Each question carries 1 mark.

1. State Cauchy's General Principle of Convergence for series.
2. Is the series $2 + \frac{3}{2} + \frac{4}{3} + \dots$ convergent? Why?
3. Define absolutely convergent series.
4. Define discontinuity of the first kind.
5. Define partition of $[a, b]$ and refinement of a partition.
6. Define intermediate value property of a function f on $[a, b]$.
7. Let $f(x) = 1$ for all x in $[0, 1]$. Is f integrable on $[0, 1]$? Why or why not?
8. State the fundamental theorem of calculus.
9. Show that the series $\sum \frac{\cos n\theta}{2^n}$ converges uniformly for all real values of θ .
10. State Abel's test.

(10 × 1 = 10)

Part B

Answer any eight questions.

2 marks each.

11. Prove or disprove : If $u_n \rightarrow 0$ then $\sum u_n$ is convergent.
12. Test the convergence of the series :

$$\frac{1}{1 \cdot 2 \cdot 3} + \frac{3}{2 \cdot 3 \cdot 4} + \frac{5}{3 \cdot 4 \cdot 5} + \dots$$

Turn over

13. State D'Alembert's ratio test.
14. Prove that every absolutely convergent series is convergent.
15. Define removable discontinuity. Illustrate it with an example.
16. If a function f is continuous on $[a, b]$ and $f(x) \in [a, b]$ for every x in $[a, b]$, prove that there exists a point c in $[a, b]$ such that $f(c) = c$.
17. Prove that $\sin x$ is uniformly continuous on $[0, \infty]$.
18. Explain upper integral and lower integral of a bounded function f on $[a, b]$.
19. Let f be a bounded function on $[a, b]$. For any partitions P_1, P_2 , prove that $L(P_1, f) \leq U(P_2, f)$.
20. Prove or disprove : If $|f|$ is integrable on $[a, b]$, then f is integrable on $[a, b]$.
21. Explain pointwise convergence of a sequence of functions.
22. Show that the sequence $\{f_n\}$ where $f_n(x) = \frac{x}{x+n}$ is uniformly convergent on $[0, k]$ for any $k > 0$.

(8 × 2 = 16)

Part C

Answer any six questions.
4 marks each.

23. Discuss the convergence of the positive term geometric series $1 + r + r^2 + r^3 + \dots$
24. State and prove Cauchy's root test.
25. State Leibnitz test. Use it to show that the series $\frac{1}{1^p} - \frac{1}{2^p} + \frac{1}{3^p} - \frac{1}{4^p} + \dots$ converges for $p > 0$.
26. Prove that a continuous function on a closed interval I is bounded on I .
27. State and prove Intermediate value theorem.
28. If f is integrable on $[a, b]$, prove that f^2 is integrable on $[a, b]$.
29. Compute $\int_{-1}^1 f dx$ where $f(x) = |x|$.
30. State and prove Cauchy criterion for uniform convergence of a sequence of functions.
31. Prove that the sequence $\{f_n\}$ where $f_n(x) = \frac{x}{1+nx^2}$, x real converges uniformly on any closed interval.

(6 × 4 = 24)

Part D

Answer any two questions.
15 marks each.

32. (a) State and prove Gauss's test.
- (b) Test for convergence the series $\sum \frac{1^2 3^2 \dots (2n-1)^2}{2^2 4^2 \dots (2n)^2} x^{n-1}, x > 0$.
33. (a) Prove that a function f defined on an interval I is continuous at a point c in I if and only if for every sequence $\{c_n\}$ in I converging to c we have $\lim_{n \rightarrow \infty} f(c_n) = f(c)$.
- (b) Give an example of a function on \mathbb{R} which is discontinuous at every point. Justify your example.
34. (a) Prove that a bounded function f is integrable on $[a, b]$ if and only if for every $\varepsilon > 0$ there exists a partition P of $[a, b]$ such that $U(P, f) - L(P, f) < \varepsilon$.
- (b) If f is a non-negative continuous function on $[a, b]$ and $\int_a^b f dx = 0$, prove that $f(x) = 0$ for all x in $[a, b]$.
35. (a) Show that the sequence $\{f_n\}$ where $f_n(x) = \tan^{-1} nx$ is uniformly convergent in any interval $[a, b], a > 0$ but is only pointwise convergent in $[0, b]$.
- (b) State and prove Weierstrass M-test.
- (c) Test for uniform convergence the series $\sum \frac{\sin n\theta}{n^2}$ for real values of θ .

(2 × 15 = 30)