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Reg. No.................................

Name.................................

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MARCH 2016

Sixth Semester

Core Course—DISCRETE MATHEMATICS

(For B.Sc. Mathematics Model I and II)

[2013 Admissions]

Time : Three Hours Maximum Marks : 80

Part A (Short Answer Questions)

Answer **all** *questions.*
Each question carries 1 *mark.*

1. Define Spanning subgraph.
2. Define a cycle.
3. Draw all trees with four vertices.
4. Find the vertex connectivity of the complete graph K_n.
5. Define a maximal non-Hamiltonian graph.
6. State Hall's marriage theorem.
7. Encrypt the message CAESAR using the Caesar cipher.
8. State Knapsack problem.
9. State absorption laws of a lattice.
10. When is an element of a lattice said to be join irreducible ?

($10 \times 1 = 10$)

Part B (Brief Answer Questions)

Answer any **eight** *questions.*
Each question carries 2 *marks.*

11. Define vertex deleted subgraph of a graph. Illustrate with an example.
12. Define component of a graph. Draw a graph with an example.
13. Prove that G is a self-complementary graph with n vertices then n is either $4t$ or $4t + 1$ for some integer t.
14. Define spanning tree of a graph. Draw two non-isomorphic spanning trees of K_4.

Turn over

15. Define cut vertex. If P_n is the path of length n, where $n \geq 3$, find the cut vertices of P_n.

16. Give an example of (i) a graph which is Eulerian but not Hamiltonian ; (ii) a graph which is not Eulerian but Hamiltonian.

17. Define a matching. When is it said to be a perfect matching ?

18. Write a brief note on personnel assignment problem.

19. Distinguish between monoalphabetic cipher and polyalphabetic cipher.

20. Encipher the message HAPPY DAYS using the auto key cipher with seed K.

21. Show that the set L of all factors of 12 under the autokey forms a lattice.

22. Show that sublattice of a modular lattice is modular.

$(8 \times 2 = 16)$

Part C (Short Essay Type Questions)

Answer any **six** *questions.*
Each question carries 4 *marks.*

23. Define adjaceny matrix and incidence matrix of a graph. Illustrate with an example.

24. Define any two vertices u and v of a graph G. Prove that every $u = v$ walk contains a $u - v$ path.

25. Let G be a graph with n vertices, where $n \geq 2$. Prove that G has at least two vertices that are not cut vertices.

26. Explain Konigsberg bridge problem and draw a graph representing the problem.

27. Prove that a simple graph G is Hamiltonian if and only if closure of G is Hamiltonian.

28. Decrypt the message RXQTGU which was enciphered by the linear cipher $C \equiv 3P + 7 \pmod{26}$.

29. Encipher the message ATTACK AT ONCE using Vigenere cipher with the key word READY.

30. Define a distributive lattice and prove that every distributive lattice is modular.

31. Prove that the dual of a lattice is a lattice.

$(6 \times 4 = 24)$

Part D (Essay Questions)

Answer any **two** *questions.*
Each question carries 15 *marks.*

32. (a) Let G be a non-empty graph with at least two vertices. If G has no odd cycles, prove that G is bipartite.

(b) Prove that a graph G is connected if and only if it has a spanning tree.

33. (a) Prove that a connected graph is Euler if and only the degree of every vertex is even.

    (b) Let G be a k-regular bipartite graph with $k > 0$. Prove that G has a perfect matching.

34. (a) Explain how encryption and decryption are carries out in RSA crypto system.

    (b) A user of the Knapsack crypto system has the sequence 49, 32, 30, 43 as listed encryption key. If the user's private key involves the modulus $m = 50$ and multiplier $a = 33$, determine the secret super increasing sequence.

35. (a) In a lattice L, prove that :

    $a \wedge (b \vee c) \geq (a \wedge b) \vee (a \wedge c)$ for all a, b, c in L.

    (b) Define a distributive lattice. Prove that the dual of a distribution lattice is distributive.

    (c) Show that a chain is a distributive lattice.

(2 × 15 = 30 marks)