**TB156495A** **Reg. No…………………………….**

**Name……………………………….**

**B. Sc. DEGREE (C.B.C.S.S) EXAMINATION, MARCH 2018**
**(2015 Admission Regular)**
**SEMESTER VI – CORE COURSE (MATHEMATICS)**
**MT6B09B -REAL ANALYSIS II**

**Time: Three Hours** **Maximum Marks: 80**

## PART A

**I. Answer all questions. Each question carries 1 mark.**

1. State Cauchy's General Principle of Convergence.
2. Define Absolute Convergence.
3. Define continuity at an internal point 'c' of a function f.
4. For any two partitions $P_1$ ,$P_2$ , Show that $L(P_1,f) \leq U(P_2,f)$.
5. Define uniform convergence of a sequence of functions.
6. Show that $\sum \frac{\cos nx}{n^2}$ is uniformly convergent for all values of $x$, where $p > 1$.

**(6X1=6)**

## PART B

**II. Answer any seven questions. Each question carries 2 marks.**

7. State and Prove Geometric Series Test.
8. Show that the series $\sum (\log n)^{-p}$ diverges for p>0.
9. Prove that a series $\sum u_n$ of positive terms converges if $\lim_{n\to\infty} (u_n)^{\frac{1}{n}}$ exists and is less than 1.
10. If a function f is continuous on a closed interval [a,b] then prove that it attains its bounds at least ones in [a,b].
11. Show that the function $f(x)=x^2$ is uniformly continuous on [-1,1]
12. Show that the function $f(x)=\frac{1}{1+|x|}$ for real x is continuous and bounded and attains its supreme for x=0 but does not attain the infimum.
13. Show that the function f defined by

    $$f(x) = \begin{cases} 0, 'x' is' rational' \\ 1, 'x' is' irrational \end{cases},$$

    is not integrable on any interval.
14. Show that if $f$ is bounded and integrable on $[a,b]$ and $k$ is a number such that $|f(x)| \leq k \ \forall x \in [a,b]$ then $\left|\int_a^b f(x)dx\right| \leq k|b-a|$.
15. State and prove Weierstrass M test.

16. Show that $\{f_n\}$ where $f_n(x) = \frac{1}{x+n}$ is uniformly convergent in $[0,b]$,b>0.

**(7X2=14)**

## PART C

**III. Answer any five questions. Each question carries 6 marks.**

17. State and Prove D'Alembert's Ratio Test.
18. Test the convergence of the series:

$$\frac{\sim}{} + \frac{1+\sim}{1+} + \frac{(1+\varpi)(2+\varpi)}{(1+\beta)(2+\beta)} + \frac{(1+\varpi)(2+\varpi)(3+\varpi)}{(1+\beta)(2+\beta)(3+\beta)} + \dots$$

19. Show that the function f defined by

$f(x) = \begin{cases} 1, if\ x\ is\ rational \\ 0, if\ xis\ irrational \end{cases}$ is discontinuous at every point.

20. State that if a function f is continuous on a [a,b] , then it attains bounds at least once in[a,b].
21. State and Prove a characterization for the integrability of a bounded function.
22. Without doing direct integration show that $\int_1^2 f dx = \frac{11}{2}$ where $f(x) = 3x+1$.
23. Consider the series $\sum_{n=0}^{\infty} f_n$ where $f_n(x)$ is defined as $\frac{x^2}{(1+x^2)^n}$, $x \in$ **R.** Find its sum function**.**
24. Show that { $f_n(x)$ } where $f_n(x) = x^n$ is uniformly convergent on $[0,k], k<1$.

**(5X6=30)**

## PART D

**IV. Answer any two questions. Each question carries 15 marks.**

25. State and Prove Gauss Test.
26. Show that if the function f is continuous on a (a,b) and f(a) and f(b) are the opposite signs , then there exists at least one point $\sim \in (a,b)$ such that $f(\sim) = 0$.
27. If a function $f$ is bounded and integrable on $[a,b]$. Prove that the function $F$ defined as

$F(x) = \int_a^x f(t)dt, a \le x \le b$ is continuous on $[a,b]$ and furthermore if $f$ is continuous at a point $c$ of $[a,b]$ ,then $F$ is derivable at $c$ and $F'(c) = f(c)$.

28. State and Prove Cauchy's criterion for uniform convergence for a sequence of functions $\{f_n\}$.

**(2×15=30)**