**TB156510A** **Reg. No…………………………….**

**Name………………………………….**

**B. Sc. DEGREE (C.B.C.S.S) EXAMINATION, MARCH 2018**
**(2015 Admission Regular)**
**SEMESTER VI – CORE COURSE (MATHEMATICS)**
**MT6B12B – LINEAR ALGEBRA**

**Time: Three Hours** **Maximum Marks: 80**

## PART A

**I. Answer all questions. Each question carries 1 mark.**

1. Determine whether the set of all diagonal $2 \times 2$ matrices under regular scalar multiplication but vector addition defined to be $A \oplus B = AB$, where $A, B \in M_{2\times 2}$, is a vector space.
2. Determine whether the set $S = \{[\sim \quad 2\sim \quad 4\sim] \mid \sim \in R\}$ is a subspace of $R^3$.
3. Is T defined by $T: R^2 \to R^2$, $T[a \quad b] = [a+2 \quad b-2]$, a linear transformation.
4. Find $T(u+3v)$ for a linear transformation if $T(u) = 22$ and $T(v) = -8$.
5. The determinant of a $2 \times 2$ matrix A is -40 and trace is 6. Find the eigen values of A.
6. Find the angle between $[2 \quad 5]$ and $[-3 \quad 4]$.

**(6x1=6)**

## PART B

**II. Answer any seven questions. Each question carries 2 marks.**

7. Determine the coordinate representation of $V = [7 \quad 2]$ with respect to the basis $S = \{[1 \quad 1], [1 \quad -1]\}$.
8. Define row space and row rank of a matrix.
9. Use row rank to determine whether $S = \{[2 \quad -4], [-3 \quad 6]\}$ is linearly independent.
10. Find the matrix representation of $T: P^2 \to P^3$, $T(at^2 + bt + c) = at^3 + bt^2 + ct$, with respect to the standard bases of $P^2$ and $P^3$.
11. Find the transition matrix from $C = \{[0 \quad 1]^T, [1 \quad 1]^T\}$ to $D = \{[1 \quad 1]^T, [1 \quad 2]^T\}$.
12. Determine the image of the matrix $A = \begin{bmatrix} 1 & 1 & 5 \\ 2 & -1 & 1 \end{bmatrix}$.
13. Find the eigen values of $A = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$.
14. Prove that similar matrices have the same characteristic equation.
15. Determine whether $X = [1 \quad 2]^T$ and $Y = [3 \quad 4]^T$ are orthogonal.
16. Find $\|X\|$ for $X = [1/4 \quad 1/2 \quad 1/8]^T$. **(7X2=14)**

## PART C

**III. Answer any five questions. Each question carries 6 marks.**

17. Find a basis for the row space of $A = \begin{bmatrix} 1 & 2 & 1 & 3 \\ 2 & 3 & -1 & -6 \\ 3 & -2 & -4 & -2 \end{bmatrix}$.

18. Determine whether $t^3 + t^2 + t$ is a linear combination of $\{t^3 + t^2, t^3 + t, t^2 + t\}$.

19. Find the matrix representation of the linear transformation $T : R^2 \to P^2$ defined by $T[a\ b]^T = (4a + b)t^2 + 3at + 2a - b$, with respect to the standard basis B of $R^2$ and the basis C = $\{t^2 + t, t + 1, t - 1\}$.

20. Find the rank and nullity of given transformation and determine whether it is one-one and onto. $T : R^2 \to R^2$, T[a b] = [a a+b].

21. Determine the eigen space of $A = \begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix}$.

22. Prove that the product of all eigen values (including multiplicity) equals the determinant of the matrix.

23. Calculate the induced inner product of p(t) = $t^2+2t + 3$ and q(t) = $t^2 + 3t - 5$ in $P^2$.

24. Prove that if *X, Y, Z* are vectors in $R^n$ then

a) $\langle X, X \rangle > 0$, for $X \neq 0$, b) $\langle X, Y \rangle = \langle Y, X \rangle$ c) $\langle \lambda X, Y \rangle = \lambda \langle X, Y \rangle$ for any real λ.

**(5X6=30)**

## PART D

**IV. Answer any two questions. Each question carries 15 marks.**

25. Prove that if S = $\{v_1, v_2 ... v_n\}$ is a basis for a vector space V, then any set containing more than n vectors is linearly dependent.

26. a) If $v_C$ and $v_D$ are the n tuple representations of v with respect to the bases C and D, the prove that $v_D = P_C^D v_C$ where $P_C^D$ is the transition matrix from C to D.

    b)Prove that the transition matrix from C to D , where both C and D are bases for the same finite dimensional vector space , is invertible and its inverse is the transition matrix from C to D.

27. Prove that eigen vectors of a matrix corresponding to distinct eigen values are linearly independent.

28. a) State and prove parallelogram law of vectors.

    b) State and prove Cauchy Scharz inequality.

    c) Prove the triangle inequality of vectors.

    d) Prove that if X and Y are orthogonal $\|X + Y\|^2 = \|X\|^2 + \|Y\|^2$. **(2X15=30)**