TB156500A **Reg. No……………………………….**

Name………………………………….

B. Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MARCH 2018
(2015 Admission Regular)
SEMESTER VI – CORE COURSE (MATHEMATICS)
MT6B10B - COMPLEX ANALYSIS

Time: Three Hours **Maximum Marks: 80**

PART A

I. Answer all questions. Each question carries 1 mark.

1. Write the domain of definition of the function $f(z) = \frac{z}{z+\bar{z}}$
2. Show that $u = x^2 - y^2$ is a harmonic function
3. State Cauchy-Goursat Theorem.
4. Define smooth arc.
5. Write the Maclaurin's series expansion of e^z.
6. Define the essential singular point of f(z).

(6x1=6)

PART B

II. Answer any seven questions. Each question carries 2 marks.

7. Show that $f(z) = e^{\bar{z}}$ is nowhere differentiable.
8. Prove that $\text{Log}(-1) = \pi i$.
9. Show that if f is a differentiable function at z_0 then it is continuous at z_0.
10. Define simply connected domain and multiply connected domain with examples.
11. Evaluate $\int_c \frac{z+2}{z}\, dz$ where c is the semicircle $z = 2e^{i\theta} (o \le \theta \le \pi)$
12. If c is the positively oriented unit circle $|z| = 1$ then find the value of $\int_c \frac{e^{2z}}{z^4}\, dz.$
13. Suppose that $z_n = x_n + iy_n$ and $z = x + iy$. Show that if $\lim_{n\to\infty} z_n = z$ then $\lim_{n\to\infty} x_n = x$ and $\lim_{n\to\infty} y_n = y$.
14. Show that if $\sum_{n=1}^{\infty} z_n = s$ then $\sum_{n=1}^{\infty} \overline{z_n} = \bar{s}$.
15. Find the residues of e^{1/z^2} at z = 0.
16. State Jordan's lemma.

(7X2=14)

PART C

III. Answer any five questions. Each question carries 6 marks.

17. Find the eighth root of $-8i$.
18. A Function f (z) = u+iv is analytic in a domain iff v is a harmonic conjugate of u.
19. Evaluate $\int_c \frac{1}{z^2+4} dz$ *, where C is the circle* $|z - i| = 2$ *in the positive sense.*
20. State and prove Liouville's theorem.

21. Derive the Taylor series representation $\frac{1}{1-z}$ in powers of $z - i$.
22. Show that when $0 < |z-1| < 2$

$$\frac{z}{(z-1)(z-3)} = -3\sum_{n=0}^{\infty}\frac{(z-1)^n}{2^{n+2}} - \frac{1}{2(z-1)}$$

23. State and prove Cauchy-residue theorem.
24. Evaluate $\int_0^{2\pi}\frac{d\theta}{5+\sin\theta}$.

(5X6=30)

PART D

IV. Answer any two questions. Each question carries 15 marks.

25. State and prove the necessary and sufficient condition for a function $f(z) = u + iv$ is analytic.
26. a. State and prove maximum modules principle.
    b. State and prove Fundamental theorem of algebra.
27. State and prove Taylor's Theorem.
28. Use residue to evaluate $\int_0^{\infty}\frac{x^2dx}{x^6+1}$.

(2X15=30)