

B. Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MARCH 2017
(2016 Admission - Regular & 2015 Admission – Supplementary / Improvement)
SEMESTER II - COMPLEMENTARY COURSE (MATHEMATICS)
MT2CPC02B – PARTIAL DERIVATIVES, MULTIPLE INTEGRALS,
TRIGONOMETRY AND MATRICES
(For Physics & Chemistry)

Time: Three Hours

Maximum Marks: 80

PART A

I. Answer all questions. Each question carries 1 mark.

1. State Fubini's Theorem.
2. Evaluate $\int_0^1 \int_0^2 (4 - x - y) dx dy$
3. Prove that $\cosh 2x = 1 + 2(\sinh x)^2$
4. Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ where $f(x, y) = \log(x^2 + y)$
5. Find the real part of $\cos(\alpha + i\beta)$
6. Define rank of a Matrix.

(6x1 = 6)

PART B

II. Answer any seven questions. Each question carries 2 marks.

7. Evaluate $\iint_R \frac{x}{y} dA$ where R is the region in the first quadrant bounded by the lines $y = x$, $y = 2x$, $x = 1$ and $x = 2$.
8. Find the average value of $f(x, y) = x \cos(xy)$ over the rectangle R: $0 \leq x \leq \pi$, $0 \leq y \leq 1$
9. Evaluate $\int_0^1 \int_0^\pi \int_0^\pi y \sin z dz dy dx$
10. Prove that $\cos 4\theta = \cos^4 \theta - 6 \cos^2 \theta \sin^2 \theta + \sin^4 \theta$
11. If $\sin(A + iB) = x + iy$ prove that $\frac{x^2}{\sin^2 A} - \frac{y^2}{\cos^2 A} = 1$
12. Separate into real and imaginary parts of the expression $\cosh(\alpha + i\beta)$
13. Find the values of $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ at the point (4, -5) if $f(x, y) = x^2 + 3xy + y - 1$
14. Find all the second order partial derivatives of $f(x, y) = xe^y + \cos(xy)$
15. Reduce to normal form the matrix $\begin{bmatrix} 1 & 2 & 1 \\ -1 & 0 & 2 \\ 2 & 1 & -3 \end{bmatrix}$
16. Obtain the row equivalent canonical matrix of $\begin{bmatrix} 1 & 2 & -3 \\ 2 & 5 & -4 \end{bmatrix}$

(7x2 = 14)

PART C

III. Answer any five questions. Each question carries 6 marks.

17. Change the order of integration and evaluate $\int_0^\pi \int_x^\pi \frac{\sin y}{y} dy dx$
18. Find the area enclosed by one leaf of the rose $r = 12 \cos 3\theta$
19. Separate into real and imaginary parts the quantity $\tan^{-1}(\alpha + i\beta)$
20. Prove that $\sin^6 \theta = \frac{-1}{32} (\cos 6\theta - 6\cos 4\theta + 15\cos 2\theta - 10)$
21. Show that $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = 0$ if $f(x, y, z) = (x^2 + y^2 + z^2)^{-1/2}$
22. Find $\frac{dw}{dt}$ if $w = xy + z$; $x = \cos t$, $y = \sin t$, $z = t$ at $t = 0$
23. Show that the system of equation is consistent and solve

$$\begin{aligned} x + y + z &= 6 \\ x - y + 2z &= 5 \\ 3x + y + z &= 8 \end{aligned}$$

24. Find the eigen values and one of the eigen vectors of the matrix $\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$

(5x6 =30)

PART D

IV. Answer any two questions. Each question carries 15 marks.

25. a) Changing to polar coordinates evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} (x^2 + y^2) dy dx$
 b) Evaluate $\int_0^{\sqrt{2}} \int_0^{3y} \int_{x^2+3y^2}^{8-x^2-y^2} dz dx dy$
26. Sum the series $\frac{1}{2} \sin \alpha + \frac{1.3}{2.4} \sin 2\alpha + \frac{1.3.5}{2.4.6} \sin 3\alpha + \dots \infty$
27. a) Draw the tree diagram for both chain rule equations for two independent variables and three intermediate variables
 b) Express $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ in terms of r and s if
 $w = x + 2y + z^2$, $x = \frac{r}{s}$, $y = r^2 + \log s$, $z = 2r$
28. State Cayley Hamilton Theorem. Using Cayley Hamilton Theorem find the A^{-1} for

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -2 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

(2x15 =30)