TB162480B **Reg. No: ……**

Name: ……

B. Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MARCH 2017
(2016 Admission - Regular & 2015 Admission – Supplementary/Improvement)
SEMESTER II - CORE COURSE (MATHEMATICS)
MT2B02B - NUMBER THEORY, CRYPTOGRAPHY & CONIC SECTIONS

Time: Three Hours **Maximum Marks: 80**

PART A

I. Answer all questions. Each question carries 1 mark.

1. Find the complete solution of the Diophontine equation $5x + 22y = 18$ whose one solution is given by $x_0 = 8$, $y_0 = -1$.
2. State Sieve of Eratosthenes.
3. Define Greatest integer function.
4. Check whether the sequence 5,13,25,42,90 is super increasing or not.
5. Find the foci and vertices of $(x^2/a^2) + (y^2/b^2) = 1$ $(a > b)$
6. Define directrix of a parabola.

(6x1=6)

PART B

II. Answer any seven questions. Each question carries 2 marks.

7. If gcd(a,b) = d then prove that gcd(a/d,b/d) = 1
8. If p is a prime and p/ab then prove that p/a or p/b.
9. Establish the formula $1 + 2 + 3 + \ldots\ldots + n = \frac{n(n+1)}{2}$ for all $n \geq 1$ by Mathematical Induction.
10. Solve the linear congruence $6x \equiv 15 \pmod{21}$.
11. Show that the functions τ and σ are both multiplicative functions.
12. If $ca \equiv cb \pmod{n}$, then prove that $a \equiv b \pmod{n/d}$ where d = gcd (c,n).
13. Explain the difference between auto key and running key.
14. Using the linear cipher $C \equiv 5P + 11 \pmod{26}$ to encrypt the message ' NUMBER THEORY IS EASY'
15. Find an equation for the hyperbola with eccentricity 3/2 and directrix x = 2
16. Sketch the region in the XY-plane whose coordinates satisfy the inequality $4y^2 - x^2 \geq 4$.

(7x2=14)

PART C

III. Answer any five questions. Each question carries 6 marks.

17. Use the Euclidean algorithm to obtain integers x and y satisfying gcd(56,72)=56x + 72y
18. For +ve integers a and b prove that gcd(a,b)lcm(a,b) = ab
19. State and prove Wilson's theorem.
20. The linear congruence $ax \equiv b \pmod{n}$ Has a solution if and only if d/b where d = gcd (a,n). If d/b then prove that it has d mutually incongruent solutions modulo n.
21. Calculate $5^{110} \pmod{131}$.

22. Decipher the message BBOTXWBZAWUVGK which was produced by the auto key cipher with seed RX.
23. Find the area of the region that lies inside the circle r =1 and outside the cardioid $r = 1-\cos\theta$.
24. Find the length of the cardioids $r = 1-\cos\theta$.

(5x6=30)

PART D

IV. Answer any two questions. Each question carries 15 marks.

25. State and prove Fundamental theorem of arithmetic.
26. Solve the system of congruence relation
    $x \equiv 2 \pmod 3$
    $x \equiv 3 \pmod 5$
    $x \equiv 2 \pmod 7$
27. Write a note on Public key.
28. The coordinate axes are to be rotated through an angle α to produce an equation for the curve $2x^2 + \sqrt{3}\,xy + y^2 = 10$ that has no cross product term. Find α and the new equation. Identify the curve.

(2x15=30)