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## B. Sc. DEGREE (C.B.C.S.S.) EXAMINATION, APRIL 2017 <br> Supplementary - 2014 Admission

SEMESTER V - CORE COURSE (COMPUTER APPLICATION)
MAT5DOE - DIFFERENTIAL EQUATIONS

## Time: Three Hours

Maximum Marks: 80

## PART A

I. Answer all questions. Each question carries 1 mark.

1. Write the necessary and sufficient condition for the exactness of the differential equation $M d x+N d y=0$
2. Define integrating factor of a differential equation.
3. Write Bernoullis equation.
4. Find the Wronskian of the functions $e^{x}, e^{-x}, e^{2 x}$.
5. Solve the differential equation.

$$
\frac{d^{2} y}{d x^{2}}-5 \frac{d y}{d x}+6 y=0
$$

6. Define a power series.
7. What is meant by regular singular point of a $2^{\text {nd }}$ order differential equation

$$
a_{0}(x) y^{\prime \prime}+a_{1}(x) y^{\prime}+a_{2}(x) y=0
$$

8. Write the Bessel's function of $1^{\text {st }}$ kind of order $p$.
9. Define partial differential equation.
10. Write the direction cosine of the normal to the surface $z=f(x, y)$

## PART B

II. Answer any eight questions. Each question carries $\mathbf{2}$ marks.
11. Solve $\left(2 x \cos y+3 x^{2} y\right) d x+\left(x^{3}-x^{2} \sin y-y\right) d y=0$
12. Solve $(x-y) y^{4} d x-x^{3}\left(y^{2}-3\right) d y=0$
13. Find the integrating factor of the differential equation

$$
\frac{d y}{d x}+\left(\frac{2 x+1}{x}\right) y=e^{-2 x}
$$

14. Find the integrating factor and solve.

$$
\left(2 x^{2}+y\right) d x+\left(x^{2} y-x\right) d y=0
$$

15. Show that $x^{2}$ and $1 / x^{2}$ are linearly independent solution of the differential equation.

$$
x^{2} \frac{d^{2} y}{d x^{2}}+x \frac{d y}{d x}-4 y=0
$$

16. Find $y$, such that $\left(D^{3}-5 D^{2}+9 D-5\right) y=0$
17. Solve $\left(x^{2} D^{2}+x D+1\right) y=0$
18. Prove that $\sqrt[n+1]{n}=n \mid n$
19. Show that $\frac{d\left(x^{p} J_{p}(x)\right)}{d x}=x^{p} J_{p-1}(x)$
20. Find the indicial equation of the differential equation $\left(x^{2}-1\right) \frac{d^{2} y}{d x^{2}}+3 x \frac{d y}{d x}+x y=0$.
21. Form a partial differential equation by eliminating the arbitrary function.

$$
z=x y+f(x+y)
$$

22. Find the integral curve of the equation. $\frac{d x}{x}=\frac{d y}{y}=\frac{d z}{z}$

## PART C

## III. Answer any six questions. Each question carries 4 marks.

23. Solve

$$
\left.\left(x^{2}+y^{2}\right) d x\right)-2 x y d y=0
$$

24. Solve $\left(2 x y^{2}+y\right) d x+\left(2 y^{3}-x\right) d y=0$
25. Find a family of oblique trajectories that intersect the family of circles $x^{2}+y^{2}=c^{2}$ at angle $45^{\circ}$.
26. Given that $y=x$ is a solution of $\left(x^{2}-x+1\right) \frac{d^{2} y}{d x^{2}}-\left(x^{2}+x\right) \frac{d y}{d x}+(x+1) y=0$. Find a linearly independent solution by reducing the order. Write the general solution.
27. Find the general solution of the equation $x^{2} \frac{d^{2} y}{d x^{2}}+4 x \frac{d y}{d x}+2 y=4 \log x$
28. Find the powerseries solution of the differential equation.

$$
\frac{d^{2} y}{d x^{2}}+x \frac{d y}{d x}+\left(x^{2}+2\right) y=0
$$

29. Show that
a. $\frac{d\left(x^{-p} / p_{p}(x)\right)}{d x}=-x^{-p} J_{p+1}(x)$
b. Express $J_{1}(x)$ and $\frac{d\left(J_{1}(x)\right)}{d x}$ in terms of $J_{0}(x)$ and $J_{1}(x)$.
30. Find the integral curves of the equation.

$$
\frac{a d x}{b c(y-z)}=\frac{b d y}{a c(z-x)}=\frac{c d z}{a b(x-y)}
$$

31. Solve the equation.

$$
(x+z) p+y q=z+y^{2}
$$

## PART D

## IV. Answer any two questions. Each question carries 15 marks.

32. Solve:
a) $\frac{d y}{d x}=\frac{x+7 y+2}{3 x+5 y+6}$
b) $\left(x \tan \frac{y}{x}+y\right) d x-x d y=0$
33. Solve $x^{2} \frac{d^{2} y}{d x^{2}}-2 x \frac{d y}{d x}+2 y=x^{3}$
34. Solve $\begin{array}{ll}\text { a) } 2 \frac{d x}{d t}-2 \frac{d y}{d t}-3 x=t & \text { b) } 2 \frac{d x}{d t}+2 \frac{d y}{d t}+3 x+8 y=2\end{array}$
35. Find the integral curves of the following equations
a) $\frac{d x}{x z-y}=\frac{d y}{y z-x}=\frac{d z}{1-z^{2}}$
b) $\frac{d x}{x^{2}\left(y^{3}-z^{3}\right)}=\frac{d y}{y^{2}\left(z^{3}-x^{3}\right)}=\frac{d z}{z^{2}\left(x^{3}-y^{3}\right)}$
