**TB145601B** **Reg. No.........................**

**Name............................**

**B. Sc. DEGREE (C.B.C.S.S.) EXAMINATION, APRIL 2017**
**Supplementary – 2014 Admission**
**SEMESTER V - CORE COURSE (COMPUTER APPLICATION)**
**CA5MA – MATHEMATICAL ANALYSIS**

**Time: Three Hours** **Maximum Marks: 80**

## PART A

**I. Answer all questions. Each question carries 1 mark.**

1. Find the supremum of the $set \left\{\frac{1}{n} - \frac{1}{m} : m, n \in N\right\}$.
2. State Archimedean Property of real numbers $\mathbb{R}$.
3. Give an example of a perfect set.
4. Give an example of a set which is neither closed nor open.
5. Define limit point of a set.
6. State Cauchy's first theorem.
7. Find $\lim_{n\to\infty} \frac{1+2+\cdots+n}{n^2}$.
8. Give an example of a nowhere dense set.
9. If $z_1 = (-2,3)$ *and* $z_2 = (3,2)$ locate $z_1 + z_2$ vectorially.
10. If $z_1 = -1$ *and* $z_2 = -i$, find Arg $(z_1 z_2)$ where Arg z denotes the principal value of the argument of z

**(10 x 1 = 10)**

## PART B

**II. Answer any eight questions. Each question carries 2 marks.**

11. State Dedekind's form of completeness property.
12. Show that a nonempty finite set is not a neighbourhood of any point
13. If $S$ *and* $T$ are subsets of real numbers then show that $(S \cap T)' \subseteq S' \cap T'$.
14. Show by an example that the union of an arbitrary family of closed sets may not always be a closed set.
15. Show that the set $\left\{1, -1, 1\frac{1}{2}, -1\frac{1}{2}, 1\frac{1}{3}, -1\frac{1}{3}, \dots\right\}$ is closed but not open.
16. Show that a convergent sequence of real numbers is bounded.
17. Show that the sequence $\{b_n\}$, where $b_n = \frac{1}{(n+1)^2} + \frac{1}{(n+2)^2} + \cdots + \frac{1}{(2n)^2}$ converges to zero.
18. Show that for any real number x, $lim \frac{x^n}{n!} = 0$.
19. Show that rational numbers in $[0,1]$ is countable.
20. Write $1 - i$ in the polar form.
21. Find the cube root of $(1 + i)$.
22. Show that $|z_1 + z_2| \leq |z_1| + |z_2|$.

**(8 x 2 = 16)**

## PART C

**III. Answer any six questions. Each question carries 4 marks.**

23. Prove that the set of limit points of a bounded sequence has the greatest and the least members.
24. Show that the interior of a set S is an open set.
25. Show that intersection of arbitrary family of closed sets is closed.
26. Show that the derived set of a set is closed.
27. Show that a countable union of countable sets is countable.
28. Prove that every bounded sequence with a unique limit point is convergent.
29. Show that the sequence $\left\{a_n^{\frac{1}{n}}\right\}$, where $a_n = \frac{3n!}{(n!)^3}$, is convergent and find its limit.
30. Prove that $|Im\,(1 - \bar{z} + z^2)| < 3$ *when* $|z| < 1$.
31. State and prove Cantor's intersection theorem.

**(6 x 4 = 24)**

## PART D

**IV. Answer any two questions. Each question carries 15 marks.**

32. Prove the equivalence of order completeness property and Dedekind's property of R.
33. (a) State and prove Bolzano Weierstrass theorem for sets.
    (b) Prove that the set of all rational numbers Q is countable.
34. (a) State and prove Cauchy's second theorem.
    (b) Show that $\lim \sqrt[n]{n} = 1$.
35. State and prove Cauchy's general principle of convergence.

**(2 x 15 = 30)**