TB145600A **Reg. No ………………………**

Name …………………………

B. Sc. DEGREE(C.B.C.S.S.)EXAMINATION, OCTOBER 2016
SEMESTER V – MATHEMATICS
MAT5MA – MATHEMATICAL ANALYSIS

Time: Three Hours **Maximum Marks: 80**

PART A

Short answer questions

I . Answer all questions. Each question carries 1 mark.

1. Find the supremum of the *set* $\left\{1+\frac{(-1)^n}{n} : n \in N\right\}$.
2. State Order Completeness Property of $\mathbb{R}$.
3. Give an example of an open set which is not an interval.
4. Find the derived set of $\left\{1+\frac{1}{n} : n \in N\right\}$.
5. Show by an example that the union of arbitrary family of closed sets may not be closed.
6. Define limit inferior of the sequence $\{a_n\}$.
7. State Cauchy's second theorem
8. Show by an example that every bounded sequence need not be convergent.
9. If $z_1 = (3,1)$ *and* $z_2 = (1,2)$ locate $z_1 - z_2$ vectorially.
10. Write $\frac{3+2i}{1-i}$ in the form a+*i*b.

(10 x 1 = 10)

PART B

II . Answer any eight questions. Each question carries 2 marks.

11. If x and y are real numbers with x$< y$ then show that there exists an irrational number z such that $x < z < y$.
12. If M and N are neighbourhoods of x, then show that $M \cap N$ is also a neighbourhood of x.
13. If $S \subseteq T$ are subsets of real numbers then show that $S' \subseteq T'$.
14. Show that the closure of a set S is closed.
15. Define perfect set. Give an example.
16. Show that $\lim_{n\to\infty} \frac{2n^2-5}{3n^2+7n} = \frac{2}{3}$.
17. Show that $\lim_{n\to\infty} \frac{1}{n}\left[1+2^{\frac{1}{2}}+3^{\frac{1}{3}}+\cdots+n^{\frac{1}{n}}\right] = 1$.
18. Show by Cauchy's general principle of convergence that the sequence$\{S_n\}$, where $S_n = 1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}$ cannot converge.
19. Show that a set is closed if and only if its complement is open.
20. Write 1-i$\sqrt{3}$in the exponential form.
21. Find the value of z satisfying the condition arg$\frac{z-1}{z+2} = \frac{\pi}{3}$.

22. Find the smallest positive integer n such that $\left(\frac{1+i}{1-i}\right)^n = 1$.

(8 x 2 = 16)

PART C

III . Answer any six questions. Each question carries 4 marks.

23. State and prove Archimedean property.
24. Show that every open set is the union of open intervals.
25. Show that union of arbitrary family of open sets is open.
26. Show that $(S \cup T)' = S' \cup T'$.
27. Show that the set of real numbers in [0,1] is uncountable.
28. State and prove Cauchy's first theorem.
29. Show that $\lim_{n\to\infty} \sqrt[n]{n} = 1$.
30. Show that $\lim_{n\to\infty} \frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + \ldots + \frac{1}{\sqrt{n^2+n}} = 1$.
31. Prove that $\sqrt{2}\,|z| \geq |Re\, z| + |Im\, z|$.

(6 x 4 = 24)

PART D

IV . Answer any two questions. Each question carries 15 marks.

32. Prove that set of all rational numbers is not order complete.
33. (a) Derived set of a bounded set is bounded.

    (b) Show that every bounded infinite set has the smallest and the greatest limit points.
34. (a)State and prove nested interval property of real numbers.

    (b)Show that the sequence $\frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{n+n}, \forall n \in \mathbb{N}$, is convergent.
35. State and prove Bolzano Weierstrass theorem for sequences. Is the converse true? Justify.

(2 x 15 = 30)