

B. Sc. DEGREE (C.B.C.S.S.) EXAMINATION, OCTOBER 2016
SEMESTER V - MATHEMATICS
MAT5FM - FUZZY MATHEMATICS

Time: Three Hours

Maximum Marks: 80

PART A**Short Answer Questions****I. Answer all questions. Each question carries 1 mark.**

1. Define support of a fuzzy set
2. Compute the scalar cardinalities for the fuzzy set given by $D(x) = 1 - \frac{x}{10}$ for $x \in \{0,1,2, \dots, 10\} = X$
3. State the second decomposition theorem
4. Define fuzzy t- norm
5. Write the "Sugeno class" of fuzzy complements
6. State first characterization theorem of fuzzy complements
7. Determine whether the fuzzy set by the following function $A(x) = \begin{cases} \sin x & 0 \leq x \leq \pi \\ 0 & \text{otherwise} \end{cases}$ is a fuzzy number
8. Calculate $[-3,4] \cdot [2,6]$
9. Define Linguistic hedges
10. Define unqualified fuzzy proposition

(10x1=10)

PART B**Brief Answer Questions****II. Answer any eight questions. Each question carries 2 marks.**

11. Show that the law of contradiction is violated for fuzzy sets.
12. For $A, B \in \mathcal{F}(X)$ and $\alpha \in [0,1]$, Show that $(A \cup B)^\alpha = A^\alpha \cup B^\alpha$.
13. Let $f: X \rightarrow Y$ be an arbitrary function. Then, for any $A \in \mathcal{F}(X)$ and all $\alpha \in [0,1]$ prove that $\alpha^+ [f(A)] = f(\alpha^+ A)$
14. Prove that the standard fuzzy intersection is the only idempotent t- norm
15. The triples $\langle \min, \max, c \rangle$ and $\langle i_{\min}, i_{\max}, c \rangle$ are dual with respect to any fuzzy complement c
16. Find the equilibrium point of $c_\gamma(a) = \frac{\gamma^2(1-a)}{a+\gamma^2(1-a)}$
17. Does distributive law hold for arithmetic operations on closed intervals?. Justify your answer
18. If $A(x) = \begin{cases} 0 & x \leq -1 \text{ and } x > 3 \\ (x+1)/2 & -1 < x \leq 1 \\ (3-x)/2 & 1 < x \leq 3 \end{cases}$ and $B(x) = \begin{cases} 0 & \text{for } x \leq 1 \text{ and } x > 5 \\ (x-1)/2 & 1 < x \leq 3 \\ (5-x)/2 & 3 < x \leq 5 \end{cases}$ then compute $A - B$
19. Prove that $X = B - A$ is not the solution of the fuzzy equation $A + X = B$

20. Define Boolean algebra on a set A
21. Define fuzzy quantifiers with examples
22. Explain existential and universal quantifications

(8x2=16)

PART C

Short Essay questions

III . Answer any six questions. Each question carries 4 marks.

23. Prove that $S(A, B) = \frac{|A \cap B|}{|A|}$
24. Consider the fuzzy sets A and B defined on the interval $X = [0,10]$ of real numbers by the membership grade functions $A(x) = \frac{x}{x+2}, B(x) = \frac{1}{1+10(x-2)^2}$. Determine the mathematical formulas and graph of the membership grade functions of \bar{A} and \bar{B}
25. State and prove second decomposition theorem
26. For all $a, b \in [0,1], i_{min}(a, b) \leq i(a, b) \leq \min(a, b)$ where i_{min} denotes the drastic intersection
27. Define fuzzy compliment. Prove that every fuzzy compliment has at most one equilibrium
28. If A, B, C are closed intervals, prove that $A \cdot (B + C) \leq A \cdot B + A \cdot C$
Also if $b, c \geq 0$ for every $b \in B, c \in C$, then prove that $A \cdot (B + C) = A \cdot B + A \cdot C$
29. Consider two fuzzy numbers A and B defined by

$$A(x) = \begin{cases} 0 & x \leq -1 \text{ and } x > 3 \\ (x+1)/2 & -1 < x \leq 1 \\ (3-x)/2 & 1 < x \leq 3 \\ 0 & x \leq 1 \text{ and } x > 5 \end{cases}$$
 and $B(x) = \begin{cases} 0 & x \leq 1 \text{ and } x > 5 \\ (x-1)/2 & 1 < x \leq 3 \\ (5-x)/2 & 3 < x \leq 5 \end{cases}$, find $\alpha_{(A+B)}$ and $\alpha_{(A-B)}$
30. Explain different types of modifiers with examples
31. Let sets of values of variables x and y be $X = \{x_1, x_2, x_3\}$ and $Y = \{y_1, y_2\}$ respectively. Assume that the proposition "if x is A then y is B" is given where $A = .5/x_1 + 1/x_2 + .6/x_3$ and $B = 1/y_1 + .4/y_2$. Give a fact expressed by the proposition " x is A", where $A' = .6/x_1 + .9/x_2 + .7/x_3$, use generalized ponens to derive a conclusion in the form " y is B"

(6x4=24)

PART D

Essay type questions

IV . Answer any two questions. Each question carries 15 marks.

32. Distinguish between crisp set and fuzzy sets
33. Let a function $c: [0,1] \rightarrow [0,1]$ satisfy axioms c2 and c4. Then prove that c satisfies c1 and c3. Also prove that c must be a bijective function
34. State and prove a necessary and sufficient condition for $A \in \mathfrak{F}(\mathbb{R})$ to be a fuzzy number
35. Explain fuzzy quantifiers with suitable examples

(2x15=30)