

**B. Sc. DEGREE (C.B.C.S.S.) EXAMINATION, OCTOBER 2016**  
**FIFTH SEMESTER - CORE COURSE (MATHEMATICS)**  
**CA5DE – DIFFERENTIAL EQUATIONS**

**Time: Three hours**

**Maximum Marks: 80**

**Part A**

**(Short Answer Questions)**

(Answer **all** questions. Each question carries **1** mark)

1. Write the necessary and sufficient condition for the exactness of the differential equation  $Mdx + Ndy = 0$
2. Define integrating factor of a differential equation
3. Write Bernoulli's equation.
4. Find the Wronskian of the functions  $e^x, e^{-x}, e^{2x}$ .
5. Solve the differential equation.

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$$

6. Define a power series.
7. What is meant by regular singular point of a 2<sup>nd</sup> order differential equation  
 $a_0(x)y'' + a_1(x)y' + a_2(x)y = 0$
8. Write the Bessel's function of 1<sup>st</sup> kind of order  $p$
9. Define partial differential equation
10. Write the direction cosine of the normal to the surface.  $z = f(x, y)$

**(10x1=10 marks)**

**Part B**

**(Brief Answer Questions)**

(Answer any **eight** questions. Each question carries **2** marks)

11. Solve  $(2x \cos y + 3x^2y)dx + (x^3 - x^2 \sin y - y)dy = 0$
12. Solve  $(x - y)y^4dx - x^3(y^2 - 3)dy = 0$
13. Find the integrating factor of the differential equation

$$\frac{dy}{dx} + \left(\frac{2x+1}{x}\right)y = e^{-2x}$$

14. Find the integrating factor and solve.  
 $(2x^2 + y)dx + (x^2y - x)dy = 0$
15. Show that  $x^2$  and  $1/x^2$  are linearly independent solution of the differential equation.

$$x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} - 4y = 0$$

16. Find  $y$ , such that  $(D^3 - 5D^2 + 9D - 5)y = 0$

17. Solve  $(x^2D^2 + xD + 1)y = 0$

18. Prove that  $\overline{n+1} \approx n!n$

19. Show that  $\frac{d(x^p J_p(x))}{dx} = x^p J_{p-1}(x)$

20. Find the indicial equation of the differential equation  $(x^2 - 1) \frac{d^2y}{dx^2} + 3x \frac{dy}{dx} + xy = 0$ .

21. Form a partial differential equation by eliminating the arbitrary function.

$$z = xy + f(x + y)$$

22. Find the integral curve of the equation.  $\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{z}$  (8x2=16 marks)

### Part C

#### Descriptive (Short Essay questions)

(Answer any six questions. Each question carries 4 marks)

23. Solve  $(x^2 + y^2)dx - 2xydy = 0$

24. Solve  $(2xy^2 + y)dx + (2y^3 - x)dy = 0$

25. Find a family of oblique trajectories that intersect the family of circles  $x^2 + y^2 = c^2$  at angle  $45^\circ$ .

26. Given that  $y = x$  is a solution of  $(x^2 - x + 1) \frac{d^2y}{dx^2} - (x^2 + x) \frac{dy}{dx} + (x + 1)y = 0$ .

Find a linearly independent solution by reducing the order. Write the general solution.

27. Find the general solution of the equation  $x^2 \frac{d^2y}{dx^2} + 4x \frac{dy}{dx} + 2y = 4 \log x$

28. Find the powerseries solution of the differential equation.

$$\frac{d^2y}{dx^2} + x \frac{dy}{dx} + (x^2 + 2)y = 0$$

29. Show that

a.  $\frac{d(x^{-p} J_p(x))}{dx} = -x^{-p} J_{p+1}(x)$

b. Express  $J_1(x)$  and  $\frac{d(J_1(x))}{dx}$  in terms of  $J_0(x)$  and  $J_1(x)$ .

30. Find the integral curves of the equation.

$$\frac{adx}{bc(y-z)} = \frac{bdy}{ac(z-x)} = \frac{cdz}{ab(x-y)}$$

31. Solve the equation.

$$(x+z)p + yq = z + y^2$$

(6x4=24 marks)

### Part D

#### (Essay type questions)

(Answer any two questions. Each question carries 15 marks)

32. Solve

a.  $\frac{dy}{dx} = \frac{x+7y+2}{3x+5y+6}$

b.  $(x \tan \frac{y}{x} + y) dx - x dy = 0$

33. Solve  $x^2 \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} + 2y = x^3$ .

34. Solve a)  $2 \frac{dx}{dt} - 2 \frac{dy}{dt} - 3x = t$  b)  $2 \frac{dx}{dt} + 2 \frac{dy}{dt} + 3x + 8y = 2$

35. Find the integral curves of the following equations.

a)  $\frac{dx}{xz-y} = \frac{dy}{yz-x} = \frac{dz}{1-z^2}$

b)  $\frac{dx}{x^2(y^3-z^3)} = \frac{dy}{y^2(z^3-x^3)} = \frac{dz}{z^2(x^3-y^3)}$