TB145610

Reg No. . . . .
Name . . . .

B.Sc.DEGREE (C.B.C.S.S) EXAMINATION, OCTOBER 2016
SEMESTER V- MATHEMATICS
MAT5DE - DIFFERENTIAL EQUATIONS

Time: Three hours **Maximum Marks: 80**

Part A
(Short Answer Questions)
(Answer all questions.Each question carries 1 mark)

1. Show that the homogeneous equation

$$(\alpha x^2 + \beta xy + \gamma y^2)dx + (\delta x^2 + \epsilon xy + \eta y^2)dy = 0$$

is exact if and only if $\beta = 2\delta$ and $\epsilon = 2\gamma$.

2. Define a homogeneous differential equation.

3. Define integrating factor of a first order differential equation.

4. Write the general form of the n-th order linear ordinary differential equation.

5. Find the general solution of $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 25y = 0$.

6. The roots of the auxiliary equation of a certain 10th-order homogeneous linear differential equation with constant coefficients are $4, 4, 4, 4, 2+3i, 2-3i,$ $2+3i, 2-3i, 2+3i, 2-3i$.Write the general solution.

7. Locate the singular points of the differential equation $(x^4 - 2x^3 + x^2)\frac{d^2y}{dx^2} + 2(x-1)\frac{dy}{dx} + x^2y = 0$.

8. Write the Bessel's equation of order p.

9. Write the general form of the first order linear partial differential equation.

10. Find the partial differential equation corresponding to $z = (x+a)(y+b)$.

($\mathbf{10 \times 1 = 10}$)

Part B
(Brief Answer Questions)
(Answer any eight questions.Each question carries 2 marks)

11. Solve the equation $\frac{dy}{dx} + 3y = 3x^2e^{-3x}$.

12. Solve the equation $(2xy+1)dx + (x^2+4y)dy = 0$.

(P.T.O)

13. Solve the equation $4xy\,dx + (x^2+1)dy = 0$.

14. Solve the equation $(x+2y+3)dx + (2x+4y-1)dy = 0$.

15. Show that the function e^x, e^{-x} and e^{2x} are linearly independent on every real interval.

16. Solve the initial value problem $\dfrac{d^2y}{dx^2} - \dfrac{dy}{dx} - 12y = 0$; $y(0) = 3, y'(0) = 5$.

17. Find the general solution of $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} - 3y = 2e^{4x}$.

18. Find the indicial equation of the differential equation $x^2\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} - (x^2 + \frac{5}{4})y = 0$ corresponding to the regular singular point $x = 0$.

19. Locate and classify the singular points of the equation $x^2(x-2)^2\dfrac{d^2y}{dx^2} + 2(x-2)\dfrac{dy}{dx} + (x+1)y = 0$.

20. Show that $\dfrac{d}{dx}[x^p J_p(x)] = x^p J_{p-1}(x)$.

21. Find the integral curves of $\dfrac{dx}{x} = \dfrac{dy}{y} = \dfrac{dz}{z}$.

22. Show that the direction cosines of the tangent at the point (x, y, z) to the conic $ax^2 + by^2 + cz^2, x + y + z = 1$ are proportional to $(by - cz, cz - ax, ax - by)$.

(8 × 2 = 16)

Part C
Descriptive (Short Essay Questions)
(Answer any six questions.Each question carries 4 marks)

23. Solve the equation $(x^3 + y^2\sqrt{x^2+y^2})dx - (xy\sqrt{x^2+y^2})dy = 0$.

24. Solve the equation $(y^2(x+1) + y)dx + (2xy+1)dy = 0$.

25. Find the value of k such that the parabolas $y = c_1x^2 + k$ are the orthogonal trajectories of the family of ellipses $x^2 + 2y^2 - y = c_2$.

26. Solve the initial value problem $\dfrac{d^2y}{dx^2} + 8\dfrac{dy}{dx} + 16y = 8e^{-2x}$; $y(0) = 2, y'(0) = 0$.

27. Given that $e^x \sin 2x$ is a solution of $\dfrac{d^4y}{dx^4} + 3\dfrac{d^3y}{dx^3} + \dfrac{d^2y}{dx^2} + 13\dfrac{dy}{dx} + 30y = 0$, find the general solution.

28. Find power series solutions in powers of x of the differential equation $\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + y = 0$.

(P.T.O)

29. Solve the system of equations

$$2\frac{dx}{dt} - 2\frac{dy}{dt} - 3x = t$$
$$2\frac{dx}{dt} + 2\frac{dy}{dt} + 3x + 8y = 2$$

30. Show by means of an example that the parametric equations of a surface need not be unique.

31. Find the partial differential equation corresponding to $z^2(1 + a^3) = 8(x + ay + b)^3$.

(6 × 4 = 24)

Part D
(Essay Type Questions)
(Answer any two questions.Each question carries 15 marks)

32. (a) Show that the transformation $v = y^{1-n}, n \neq 0$ or 1 reduces the Bernoulli equation $\frac{dy}{dx} + P(x)y = Q(x)y^n$ to a linear equation in the dependent variable v and the independent variable x

(b) Solve the initial value problem $\frac{dy}{dx} + \frac{y}{2x} = \frac{x}{y^3}; y(1) = 2$.

33. Solve the equation $x^3\frac{d^3y}{dx^3} - x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 2y = x^3$.

34. Using the method of Frobenius find two linearly independent solutions near $x = 0$ of the differential equation $2x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - 1)y = 0$.

35. (a) Eliminate the arbitrary function f from the equation $f(x^2 + y^2 + z^2, z^2 - 2xy) = 0$ and hence find the corresponding partial differential equation.

(b) Find the general integral of the linear partial differential equation $x^2p + y^2q = (x + y)z$.

(2 × 15 = 30)