TB153520A **Reg. No: ……………………....**

Name: ………………………...

B. Sc. DEGREE (C.B.C.S.S.) EXAMINATION, OCTOBER, 2016

SEMESTER III – COMPLEMENTARY COURSE (STATISTICS)

ST3CMP03B – PROBABILITY DISTRIBUTIONS

(Common for MATHEMATICS and PHYSICS)

Time: Three Hours **Maximum Marks: 80**

Use of Scientific calculators and Statistical tables are permitted

PART A (Short Answer Questions)

I. Answer all questions. Each question carries 1 mark.

1. Define mathematical expectation.
2. Express mean of a random variable in terms of moment generating function (m.g.f) $M_X(t)$.
3. For which value of the parameter *p*, the skewness of Binomial distribution is zero?
4. Write down the p.d.f. of *X* if its m.g.f. e^{t+2t^2}.
5. Find the area between -1.96 and 1.96 under the standard normal curve.
6. If *X* is a r.v. with mean 5 and standard deviation 2, find an upper bound to $P(|X-5|>4)$.

(6x1 = 6)

PART B (Brief Answer Questions)

II. Answer any seven questions. Each question carries 2 marks.

7. If *X* and *Y* are any two random variables, show that $E(X+Y)=E(X)+E(Y)$.
8. Define moment generating function of a random variable and state any two of its properties.
9. Find *a* and *b* if $Y=aX+b$ has mean 6 and variance unity, where *X* is a random variable with mean 8 and variance 16.
10. From an urn containing lots numbered 1 to 10 one is randomly chosen. Let *X* denotes the number drawn. Write down the p.m.f. of *X* and its mean.
11. If *X* has a binomial distribution with mean 6 and variance 3.6, find $P(X=0)$.
12. Obtain the m.g.f. of exponential distribution with parameter ª .
13. If *X* is normal random variable with mean 5 and variance 9, find the distribution of $Y=2X+5$.
14. Find the mean of the Beta distribution of the first kind.
15. Define convergence in probability.
16. State Lindberg-Levy central limit theorem. Give an example.

(7x2 = 14)

PART C (Short Essay Questions)

III. Answer any five questions. Each question carries 6 marks.

17. Distinguish between raw moments and central moments. Obtain the general expression for the *r*-th central moment in terms of raw moments.
18. Find the m.g.f. of the distribution with p.d.f. $f(x) = 2x, 0 < x < 1$ and hence determine the mean and variance.
19. Derive the recurrence relation for the central moments of a Poisson distribution.
20. Define geometric distribution. State and prove the lack of memory property of geometric distribution.
21. Suppose that during a rainy season on a tropical island the length of the shower has an exponential distribution with parameter $\lambda = 2$, time being measured in minutes. i) What is the probability that a shower will last more than 3 minutes? ii) If a shower has already lasted for 2 minutes, what is the probability that it will last for at least one more minute?
22. Obtain the p.d.f. of $Y = X^2$, if X is a standard normal variable. Identify the distribution of Y.
23. State and prove Weak law of large numbers.
24. The lifetime a certain brand of an electric bulb may be considered as a random variable with mean 1200 hours and standard deviation 250 hours. Find the probability, using central limit theorem, that the average lifetime of 100 bulbs exceeds 1250 hours.

(5x6= 30)

PART D (Essay Questions)

IV. Answer any two questions. Each question carries 15 marks.

25. Let $f(x, y) = 2, 0 \le x < y \le 1$ be the joint *p.d.f.* of (X, Y). Find (i) $E(Y | X)$, (ii) $E(X | Y)$, (iii) the correlation between X and Y.
26. The screws produced by a certain machine were checked by examining samples of size 10. Fit a binomial distribution to the following data related to the number of defectives. Also find the theoretical frequencies.

No. of defectives	0	1	2	3	4	5
No. of samples	7	15	25	10	5	3

27. (a) Derive the formula for finding the central moments of Normal distribution.
    (b)In an intelligence test administered to 1000 children, the average score is 42 and standard deviation is 24. Assuming the scores obtained by the children follows normal distribution, find the expected number of children exceeding the score 60.
28. (a)State and Prove Tchebyshev's Inequality.
    (b) Two unbiased dice are thrown. If X is the sum of the numbers showing up, prove that $P(|X - 7| > 3) < 35/54$.

(2x15 = 30)