TB153440A **Reg. No…………………………….**

Name………………………………..

B. Sc. DEGREE (C.B.C.S.S.) EXAMINATION, OCTOBER 2016

SEMESTER III – CORE COURSE (MATHEMATICS)

MT3B03B - CALCULUS

Time: Three Hours **Maximum Marks: 80**

PART A

Short Answer Questions

I. Answer all questions. Each question carries 1 mark

1. Find $\frac{dy}{dx}$ if x=cosθ,y=cos nθ.
2. Write down the Maclaurin's series expansion of f(x)
3. Find $\frac{\partial f}{\partial x}$ at(4,-5), if $f(x,y)=x^2+3xy+y-1$
4. Find $\int_{2\pi}^{3\pi} 3\cos^2 x \sin x\, dx$
5. Evaluate $\int_{-1}^{1}\int_{0}^{2}(1-6x^2y)dxdy$
6. Evaluate $\int_{0}^{1}\int_{0}^{1-z}\int_{0}^{2} dxdydz$

(6x1=6)

PART B

Brief Answer Questions

II. Answer any seven questions. Each question carries 2 marks.

7. What is a point of inflexion? Show that the curve $y=x+e^x$ has no point of inflexion.
8. Find the n^{th} derivative of $y=\cos^3 x$.
9. Define Saddle Point. Show that $f(x,y)=y^2-x^2$ has a saddle point at (0,0)
10. Use the chain rule to find the derivative of $w=x^2y-y^2$ with respect to t along the path x =sint and $y=e^t$. What is the derivative's value at t=0?
11. Find the local extreme values of the function: $f(x,y)=xy-x^2-y^2-2x-2y+4$
12. Describe the method of Lagrange Multipliers for finding the local maximum and minimum values of f(x, y, z) subject to the constraint g(x, y, z)=0.
13. Find the circumference of a circle of radius r defined parametrically as x=r cost,y=r sint $0\leq t \leq 2\pi$
14. Find the area of the region between $y=3x^2$ and the x-axis on the interval [0,b].
15. Evaluate: $\int_{0}^{1}\int_{0}^{1}\int_{0}^{1}(x^2+y^2+z^2)dzdydx$
16. Write the Jacobian determinant of x=g(u,v),y=h(u,v)

(7x2=14)

PART C

Descriptive Short Essay Questions

III. Answer any five questions. Each question carries 6 marks.

17. Find y_n , if $y = \frac{x^2}{(x+2)(2x+3)}$

18. Find the ranges of values of x for which the curve $y=x^4-6x^3+12x^2+5x+7$ is concave upwards or downwards. Also determine the points of inflexion.
19. Using chain rule express $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ in terms of r and s if $w=x+2y+z^2$, $x=\frac{r}{s}$, $y=r^2+\log s$ and $z=2r$.
20. Find $\frac{\partial w}{\partial x}$ at the point $(x,y,z)=(2,-1,1)$ if $w=x^2+y^2+z^2$ and $z^3-xy+yz+y^3=1$
21. Find the volume of the solid generated by revolving the region bounded by $y=\sqrt{x}$ and the lines $y=1, x=4$ about the line $y=1$
22. Find the length of the asteroid $x=\cos^3 t, y=\sin^3 t$, $0\leq t \leq 2\pi$
23. Sketch the region of integration for the integral $\int_0^b \int_0^{\frac{a}{b}\sqrt{b^2-y^2}} xy\, dx\, dy$ and write an equivalent integral with the order of integration reversed.
24. Find the area enclosed by the cardioid , $r = a(1+\cos\theta)$.

(5x6=30)

PART D

Long Essay type Questions

IV. Answer any two questions. Each question carries 15 marks.

25. (i) State and prove Leibnitz Theorem for finding the n^{th} derivative of the product of two functions.
    (ii) If $y= \cos (m \sin^{-1}x)$, show that $(1-x)^2 y_{n+2} -(2n+1)xy_{n+1}+(m^2-n^2)y_n=0$
26. The plane $x + y + z =1$ cuts the cylinder $x^2+y^2=1$ in an ellipse. Find the points on the ellipse that lie closest to and farthest from the origin.
27. Find the length of the curve $y=\left(\frac{x}{2}\right)^{\frac{2}{3}}$ from $x=0$ to $x=2$.
28. Evaluate: $\int_0^4 \int_{\frac{y}{2}}^{\frac{y}{2}+1} \frac{2x-y}{2}\, dx\, dy$ by applying the transformation $u=\frac{2x-y}{2}$, $v=\frac{y}{2}$ and integrating over an appropriate region in the uv-plane.

(2x15=30)