B. Sc. DEGREE (C.B.C.S.S.) EXAMINATION, OCTOBER 2016

SEMESTER III - CORE COURSE (COMPUTER APPLICATION)

CAM3B03TB - CALCULUS

Time: Three Hours   **Maximum Marks: 80**

PART A

I. Answer all questions. Each question carries 1 mark

1. State Lebinitz's theorem.
2. Define saddle points
3. Write the shell formula for finding the volume of the solid generated by the revolution of a region about y-axis.
4. State Papus theorem for surface area.
5. Write the formula for surface area in differential form.
6. State Fubini's theorem first form.

                                                                                            (6 x 1 = 6)

PART B

II. Answer any seven of the following. Each question carries 2 marks

7. Differentiate the equation n times $(1 - x^2)y_2 - xy_1 - y = 0$.

8. Expand $y = e^x$ using Taylor's theorem around x =1

9. Find the point of inflexion on the curves $x = a\tan t$, $y = a\sin t\cos t$

10. If $u = xy/(x + y)$. Show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$

11. Verify that $\frac{\partial^2 u}{\partial x \partial y} \neq \frac{\partial^2 u}{\partial y \partial x}$ if $u = \log\frac{x^2+y^2}{xy}$

12. Examine $xy + \frac{1}{x} + \frac{1}{y}$ for maxima and minima.

13. Evaluate $\int_0^1 \int_0^2 xy(x - y)dy\, dx$.

14. Find the perimeter of the cardioids $r = a(1 + \cos\theta)$

15. Find the volume of the solid of cross sectional area $A(x) = 2x$ from x = 0 to x = 2.

16. Evaluate $\int_0^{\frac{\pi}{4}} \tan x \sec^2 x\, dx$.

                                                                                            (7×2=14)

PART C

III. Answer any five of the following. Each question carries 6 marks

17. Using Maclaurin's series, obtain the expansion of $\sin x$ and $\sin^{-1}x$.

18. Show that for any curve $r = f(\theta)$ the curvature is given by $\left(u + \frac{d^2u}{d\theta^2}\right)\sin^3\emptyset$, where $u = {}^1\!/_r$.

19. If v= 1/r and $r^2 = x^2+y^2$. Prove that $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = \frac{1}{r^3}$.

20. Use the method of Legrange multipliers to find the maximum value of xy subject to the constraint $2x + 2y = 16$

21. Find the area of the region in the first quadrant that is bounded above by $y = \sqrt{x}$ and below by the x axis and the line $y = x - 2$

22. Use shell method to find the volume of the solid generated by revolving the region bounded by $y = 2x - 1$, $y = \sqrt{x}$, $x = 0$

23. Evaluate $\int_0^1 \int_y^1 x^2 e^{xy}\, dx\, dy$.

24. Use shell method to find the volume of the solid generated by revolving the region bounded by $y = \frac{1}{x}$, $y = 0$, $x = \frac{1}{2}$, $x = 2$

(5×6=30)

PART D

IV. Answer any two of the following. Each question carries 15 marks

25. (a) Find the nth derivative of $= \frac{x}{x^2 + a^2}$.

   (b) Find the envelope of the curve $\left(\frac{x}{a}\right)^m + \left(\frac{y}{b}\right)^m = 1$ when $a^n + b^n = c^n$.

26. (a) Find the maximum and minimum values of the function $f(x, y) = 3x + 4y$ on the circle $x^2 + y^2 = 1$.
   (b) Find and classify the local extreame values of $f(x,y) = 3x^2 - 2y^3 - 3x^2 + 6xy$

27. The region bounded by the curve $y = x^2 + 1$ and the line $y = -x + 3$ is revolved about the x axis to generate a solid. Find the volume of the solid.

28. Find the area of the surface generated by revolving $y = \sqrt{x + 1}$, $1 \quad x \quad 5$ about x axis.

(2 x 15 = 30)