

B. Sc. DEGREE (C.B.C.S.S.) EXAMINATION, OCTOBER 2016
SEMESTER III - CORE COURSE (COMPUTER APPLICATION)
CAM3B03TB - CALCULUS

Time: Three Hours

Maximum Marks: 80

PART A**I. Answer all questions. Each question carries 1 mark**

1. Find third differential coefficient of $y = \cos(ax + b)$.
2. State Taylor's theorem
3. Define partial derivative with respect to x.
4. Define area of a closed bounded region R using a double integral.
5. Write the equation of a sphere of radius 5 with centre at origin, in spherical co-ordinates.
6. Define the volume of a closed bounded region D in space using triple integral.

(6 x 1 = 6)

PART B**II. Answer any seven of the following. Each question carries 2 marks**

7. If $y = (\sin^{-1} x)^2$ Prove that $(1 - x^2)y_2 - xy_1 = 2$.
8. Expand $\sinh x$ using Maclaurin's theorem.
9. Define Evolutes and Involutives.
10. Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ at the point (4, -5) if $f(x, y) = x^2 + 3xy + y - 1$.
11. State second derivative test for Local extreme values.
12. Find $\left(\frac{\partial w}{\partial x}\right)_{y,z}$ if $w = x^2 + y - z + \sin t$ and $x + y = t$.
13. Evaluate $\int_0^1 \int_0^{1-y} \int_0^2 dx dz dy$.
14. Find the volume of the solid generated by revolving the region under the curve $y = \sqrt{x}$, between the lines $x = 1$ and $x = 4$, about the x axis.
15. Find the area of the region enclosed by the curve $x = y^2$ and the line $x = y + 2$, integrating with respect to y.
16. Evaluate $\int_0^1 t^3 (1 + t^4)^3 dt$.

(7×2=14)

PART C**III. Answer any five of the following. Each question carries 6 marks**

17. State and prove Leibnit's theorem on the n^{th} derivative of the product of two functions.
18. (a) Find $D^n(x^2 e^x \cos x)$
 (b) Find the radius of curvature at the pole for the curve $r = a \sin n\theta$.
19. If $f(x, y) = x \cos y + ye^x$, find $\frac{\partial^2 f}{\partial x^2}$, $\frac{\partial^2 f}{\partial y \partial x}$, $\frac{\partial^2 f}{\partial y^2}$ and $\frac{\partial^2 f}{\partial x \partial y}$.
20. Find the maximum and minimum values of the function $f(x, y) = 3x + 4y$ on the circle $x^2 + y^2 = 1$.

21. The region bounded by the curve $y = \sqrt{4x - x^2}$, the x axis, and the line $x = 2$ is revolved about the x axis to generate a solid. Find the volume of the solid.
22. Applying the surface area formula find the area of the surface generated by revolving the circle $x^2 + y^2 = a^2$, about the x axis
23. Use polar co-ordinates to evaluate $\int_{-1}^1 \int_0^{\sqrt{1-x^2}} (x^2 + y^2) dy dx$
24. Find the length of the curve $y = x^{3/2}$ from $x = 0$ to $x = 4$.

(5×6=30)

PART D

IV. Answer any two of the following. Each question carries 15 marks

25. (a) If $y = \cos(m \sin^{-1} x)$. Show that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} + (m^2 - n^2)y_n = 0$.
- (b) Find the evolute of the astroid $x = a \cos^3 \theta, y = a \sin^3 \theta$.
- (c) Show that for the curve $s^2 = 8ay, \rho = 4a \sqrt{1 - \frac{y}{2a}}$.
26. (a) Find the absolute maximum and minimum values of $f(x, y) = 2 + 2x + 2y - x^2 - y^2$
- (b) Find $\frac{\partial w}{\partial x}$ at the point $(x, y, z) = (2, -1, 1)$ if $w = x^2 + y^2 + z^2, z^3 xy + yz + y^3 = 1$.
- (c) Find the local extreme values of the function $f(x, y) = xy - x^2 - y^2 - 2x - 2y + 4$.
27. Find the length of the curve $y = \left(\frac{x}{2}\right)^{2/3}$ from $x = 0$ to $x = 2$
28. The region bounded by the curve $y = x^2 + 1$ and the line $y = -x + 3$ is revolved about the x axis to generate a solid. Find the volume of the solid.

(2 x 15 = 30)