

TB154465A

Reg. No: .....

Name: .....

**B. Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MARCH 2017**  
**SEMESTER IV - COMPLEMENTARY COURSE (STATISTICS)**  
**ST4CMP04B - STATISTICAL INFERENCE**  
**(Complementary for Mathematics, Physics)**

**Time: Three Hours**

**Maximum Marks: 80**

*Use of Scientific Calculators and Statistical Tables are permitted.*

**PART A**

**I. Answer all questions. Each Question carries 1 mark.**

1. What is the square of a 't' statistic with 'n' degrees of freedom?
2. Is point estimator unique?
3. If  $E(\hat{\mu}) < \mu$ , then what is the amount of bias in the estimation set up?
4. Distinguish between null hypothesis and alternate hypothesis.
5. What is mean by p value of a test?
6. Which test is to be used for testing the hypothesis that a population mean has a given value if all the supporting parameters are known?

**(6x1=6)**

**PART B**

**II. Answer any seven questions. Each Question carries 2 marks**

7. Obtain mean of the  $t^2$  distribution with (n-1) degrees of freedom.
8. Define F statistic.
9. What are the desirable properties of a good estimator?
10. Give the interval estimate of a proportion when large samples are made from Normal distribution.
11. Estimate  $\mu$  in the density function  $f(x, \mu) = (1 + \mu)^{-x}$ ,  $0 < x < 1$  by the method of moments.
12. State Neymann-Pearson fundamental lemma for testing a simple hypothesis against a simple alternative.
13. A coin is tossed 6 times the hypothesis  $H_0: p=1/2$  is rejected if the number of heads is greater than 4. Find the probability of type 1 error and also find the probability of type 2 error; the alternative hypothesis is  $H_1: p=3/4$ .
14. Which are the main steps in solving a testing of hypothesis problem?
15. Write the test statistic for testing the hypothesis that the population mean has a specified value  $\mu_0$  from  $N(\mu, \sigma^2)$ ,  $\sigma^2$  unknown.
16. Establish the relationship between t distribution and normal distribution.

**(7x2=14)**

## PART C

### III. Answer any five questions. Each Question carries 6 marks

17. A sample of size 25 is drawn from a Normal population has variance 6.25. Find  $k$  such that  $P[\bar{X} - \mu < k] = 0.60$  where  $\bar{X}$  is the sample mean and  $\mu$  is the population mean.
18. Define  $t$  statistic. Derive two ' $t$ ' statistic by taking samples from Normal population with finite mean.
19. Examine whether sample variance is an unbiased estimate of the population variance for a normal population. If not suggest an unbiased estimate for the population variance.
20. a) Let  $x_1, x_2, \dots, x_n$  be a random sample from a Uniform population on  $(0, \theta)$ . Find sufficient estimator for  $\theta$ .  
b) If  $T$  is a consistent estimator of  $\theta$ , then find the consistent estimator of  $a\theta + b$ , where  $a$  and  $b$  are constants.
21. Show that for the normal distribution with zero mean and variance  $\sigma^2$ , the best critical region for  $H_0: \sigma = \sigma_0$  against the alternative  $H_1: \sigma = \sigma_1$  is of the form  $\sum_{i=1}^n X_i^2 \leq a_r$  for  $\sigma_0 > \sigma_1$  and  $\sum_{i=1}^n X_i^2 \geq b_r$  for  $\sigma_0 < \sigma_1$ .
22. A sample of 5 values is taken from the product of a company. The percentage of defectives in the total product is  $100p$ . To test the hypothesis  $H_0: p = 0.25$ , the following procedure is proposed; Accept  $H_0$  if the number of defectives in the sample is 0 or 1. What is 1) the critical region 2) the level of significance 3) the power of the test; if the alternative is  $H_1: p = 1/3$ .
23. Explain the procedure of testing equality of means when samples are drawn from normal population (considering all the cases).
24. In a cross-breeding experiment with plants of a certain species, 240 plants were classified into 4 classes with respect to the structure of their leaves as follows:

Class	:	I	II	III	IV	Total
Frequency:		21	127	40	52	240

According to the theory of heredity, the frequencies of the four classes must be in the ratio 1:9:3:3. Are these data consistent with the theory?

(5x6=30)

## PART D

### IV. Answer any two questions. Each Question carries 15 marks

25. a) Obtain mode of chi square distribution with  $n$  degrees of freedom.  
b) The lifetime of a certain brand of an electric bulb may be considered as a random variable with mean 1200 hours and s.d = 250 hours. Find the probability that the average lifetime of 60 bulbs exceeds 1400 hours.  
c) Derive the limiting distribution of ' $t$ ' variable with  $n$  degrees of freedom.

26. a) Let  $X_1, X_2, \dots, X_n$  be a random sample of size  $n$  from a geometric distribution for which  $p$  is the probability of success, use the following data to give a point estimate of  $p$ .

3      34    7      4      19    2      1      19    43    2  
 22     4     19     11    7      1      2      21    15    16

- b) Find the maximum likelihood estimates of  $\mu$  &  $\sigma^2$  if a random sample of size 15 from  $N(\mu, \sigma^2)$  yielded the following values

31.5   36.9   33.8   30.1   33.9   35.2   29.6   34.4  
 30.5   34.2   31.6   36.7   35.8   34.5   32.7

27. a) In a Bernoulli distribution with parameter  $p$ ,  $H_0: p=1/2$  against  $H_1: p=2/3$  is rejected if more than 3 heads are obtained out of 5 throws of a coin. Find the probabilities of type 1 error and type 2 errors.  
 b) Obtain the most powerful test for testing the mean  $\mu = \mu_0$  against  $\mu = \mu_1 (\mu_1 > \mu_0)$  when  $\sigma^2=1$  in a Normal population  $(\mu, \sigma^2)$ .

28. a) For the data in the following table, test for independence between a person's ability in Mathematics and interest in Economics.

	Ability in Maths			
	Low	Avg.	High	
Interest in Economics	Low	63	42	15
	Avg.	58	61	31
	High	14	47	29

- b) In a random sample of 500 men from a particular district of a state, 300 are found to be smokers. In one of 1000 men from another district 550 are smokers. Do the data indicate that two districts are significantly different w.r.t the prevalence of smoking among men?

**(2x15=30)**